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Spectrum Sharing Games of Network Operators and Cognitive Radios

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Summary. Since the beginning of the 20th century, the wireless frequency spectrum has been carefully controlled by government regulators. In response to the recent advances in radio technology, the spectrum regulators have opened some parts of the available spectrum for unlicensed usage. In addition, they have reformed the traditional *command and control* regulation policies and have allowed more opportunistic transmissions over unused spectrum bandwidth in licensed bands, for certain times and locations. This paradigm shift can lead to a more flexible and efficient *spectrum sharing* in the near future. In this chapter, we address the problem of spectrum sharing between network operators and cognitive radios. Because of the dynamic nature of spectrum sharing, it is difficult to analyze and to provide sound spectrum management schemes. Several researchers rely on *game theory* that is an appropriate tool for modelling strategic interactions between rational decision-makers (e.g., spectrum sharing in wireless networks). We present a selected set of works to highlight the usefulness of game theory in solving the main problems in this field.

1.1 Introduction

Wireless communications rely on the frequency spectrum as a fundamental resource. As the number of wireless communication technologies and the number of wireless networks using them kept increasing, the regulation of the access to the available frequency spectrum, i.e., controlled *spectrum sharing*, has become unavoidable. A straightforward solution for the spectrum sharing problem is to let government agencies, such as the FCC in the USA, allocate communication frequencies to different wireless networks. This was first practiced, and basically still is, on a first-come-first-served basis and then by auctions [1]. The allocated right, called the *spectrum license*, grants an exclusive usage of a given frequency band to a certain company for a given purpose. The main problem with the licensed spectrum is that the licenses are typically established for long periods of time. Recent performance studies [28,29] have shown that this significantly affects efficiency.

Around 1980, government agencies realized that the available spectrum was scarce and reserved certain frequencies as *unlicensed bands* for common use. Unlicensed bands eliminated the lengthy process of spectrum licensing thus allowing companies to enter into the communication market quickly. In spite of the advance of technologies, unlicensed bands are useless when used arbitrarily. Hence, government agencies limited the transmission power of wireless devices in unlicensed bands (this limit can vary among technologies). Yet, unlicensed bands can be quickly saturated, which also means that, in contrast to licensed usage, the quality-of-service (QoS) is hardly guaranteed by these networks. Although unlicensed bands have improved the overall spectrum utilization, they still do not solve the inflexibility caused by the licensing process.

Cognitive radio [9,16,20] is an emerging technology that enables devices to determine which of the available frequencies are unused, and to use them even if they are licensed to others. Cognitive radio devices can adapt to the actual frequency utilization, thus increasing the efficiency of wireless communication. One fundamental requirement of these devices is that they should not hamper the communication of the *primary users*, who obtained the license for the given frequency band. There exist several techniques that are appropriate for studying the behavior of this new networking environment. On the one hand, game and auction theory are useful tools to study the strategic behavior of network participants; on the other hand, graph coloring techniques can be used to assess the system optimum solution in many problems, such as the channel allocation problem. In Section 1.2 we provide a short introduction to these analytical tools.

Many researchers are currently engaged in designing spectrum sharing schemes using these analytical tools. Our goal in this chapter is to present a selected set of contributions in this field and to provide a better understanding of the current research efforts in this field. So we give only a high-level overview of the schemes. It is, of course, impossible to mention all the results of the selected papers in the field. Hence, we will briefly express the game model and the main results of each scheme.

Table 1.1 shows that we can divide the spectrum sharing games into three main groups, according to the players of the games: *licensed band operators*, *unlicensed band wireless systems*, and *cognitive radios*. In Section 1.3, we focus on the interaction between wireless operators in shared spectrum competing for users. The second set of scenarios, presented in Section 1.4, addresses the problem of unlicensed spectrum sharing. Finally, the scenarios presented in Section 1.5 are related to spectrum sharing by cognitive radios. For each game, we identify the key ideas and the game model. We discuss the main results of the games in each subsection as well.

Section	*Spectrum Sharing Game Scheme*	*Players*	*Strategy*	*Results*
1.3.1	Asymmetric Network Operators [30]	WAN and WiFi Operators	Operator Selection	Nash Equilibrium
1.3.2	National Border Spectrum Sharing [7]	Cellular Operators	Pilot Power	Nash Equilibrium with Convergence
1.3.3	Network Operators' Spectrum Sharing [8]	Cellular Operators	Pilot Power	Nash Equilibrium
1.4.1	Heterogeneous Wireless Systems [5]	High and Low power Wireless Systems	Power Spectral Density	Pareto Optimality
1.4.2	WiFi Operators' Spectrum Sharing [13]	WiFi Operators	Channel Selection	Nash Equilibrium
1.5.1	Opportunistic Spectrum Sharing [4, 26, 31, 32]	Cognitive Radios	Channel Selection	Equilibrium
1.5.2	Auction-Based Spectrum Sharing [17]	Cognitive Radios	SNR and Power	Nash Equilibrium and Pareto Optimal
1.5.3	Multi-Cell OFDMA Spectrum Sharing [14]	Cognitive Radios	Rate	Nash Equilibrium by Virtual Referee

Table 1.1. Spectrum sharing games presented in this chapter divided to three main groups: *licensed band*, *unlicensed band*, and *cognitive radio*.

1.2 Theoretical Background

Game theory, auction design, and graph coloring are the main tools for the analysis of the spectrum sharing schemes presented in this chapter. In this section, using a practical example, we introduce the fundamental concepts of non-cooperative game theory, such as Nash equilibrium (NE) and Pareto-optimality. The interested reader can find a comprehensive tutorial on game theory for wireless networks in [3, 6]. Then, we examine the benefits of using auctions in spectrum assignment and spectrum sharing design. Finally, we briefly discuss graph coloring techniques.

1.2.1 Game Theory

Game theory [10, 12, 25] is a discipline for modelling situations in which decision-makers have to make specific actions that have mutual, possibly conflicting, consequences. There is a significant amount of work in wireless networking that makes use of game theory. The basic elements of a game G are the *players*, the *strategies*, the *payoffs*, and the *knowledge* and can be shown in *strategic form* by $G = (\mathcal{P}, \mathcal{S}, \mathcal{U})$. $\mathcal{P}$, $\mathcal{S}$, and $\mathcal{U}$ are the set of all *players*, the joint set of the *strategy* spaces, and the set of *payoff* functions of all players, respectively. Considering a player $i \in \mathcal{P}$, $-i$ represents all the players belonging to $\mathcal{P}$ except i himself, they are often designated as being the *opponents* of i. S_i corresponds to the *strategy space* of player i and the set of chosen strategies constitutes a *strategy profile* s (e.g, $s = \{s_1, s_2\}$ for two players). The *payoff* $u_i(s)$ is the difference of the benefit b and the cost c of player i given the strategy profile s (i.e., $u(s) = b(s) - c(s)$). Note that, we refrain from using the word "utility" in this chapter to avoid confusion: in game theory, the utility usually corresponds to what we call the payoff in this paper, whereas in many computer science papers, the utility corresponds to what we call here the benefit.

In the following, a game that we call the *Multiple Access Game* is used to illustrate the fundamental concepts of game theory. We choose this game as a simple illustration of spectrum sharing games. In this game, two players, p_1 and p_2, share the wireless medium and want to send one packet to their respective receivers, r_1 and r_2. The players have a packet to send in each time slot, and they can either transmit (T) or stay quiet (Q). When player p_1 transmits, it incurs a transmission cost of $0 < c << 1$. The packet transmission is successful if p_2 does not transmit in the same time slot, otherwise we say that there is a collision and each players must pay the transmission costs c while the two packets are lost. If there is no collision, player p_1 gets a benefit of 1 for the successful packet transmission. We call this game a *static game* or *one-shot game* because the players have only one move to act.

Best Response

The *Multiple Access Game* can be represented in a strategic form as shown in Table 1.2. If player p_1 transmits, then the best response of player p_2 is to be quiet. Conversely, if player p_2 is quiet, then p_1 is better off transmitting a packet. We can write the best response of player i to an opponent's strategy vector s_{-i} as follows.

$p_1 \backslash p_2$	Q	T
Q	(0,0)	(0,1-c)
T	(1-c,0)	(-c,-c)

Table 1.2. The Multiple Access Game in strategic form. $(1-c, 0)$ means that player p_1's payoff is $1-c$, while player p_2 gets nothing.

***Definition*:** The *best response* $br_i(s_{-i})$ of player i to the profile of strategies s_{-i} is a strategy s_i such that:

$$br_i(s_{-i}) = \arg\max_{s_i \in S_i} u_i(s_i, s_{-i}) \tag{1.1}$$

Nash Equilibrium

If two strategies are mutually best responses to each other, then the players have no reason to deviate from the given strategy profile. In our example, two strategy profiles have this property: (Q, T) and (T, Q). To identify such strategy profiles in general, Nash introduced the concept of *Nash equilibrium* (NE) in his seminal paper [23]:

***Definition*:** The pure strategy profile s^* constitutes a Nash equilibrium if, for each player i,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \forall s_i \in S_i \tag{1.2}$$

This means that in a NE, none of the users can unilaterally change his strategy to increase his payoff. Alternatively, a NE is a strategy profile comprised of mutual best responses of all the players. Hence, the system is stable. (Q, T) and (T, Q) are two NE for *Multiple Access Game*.

The first step towards solving a game is to investigate the *existence* of NE. It is worth mentioning that Nash [12, 23, 24] proved that every finite strategic-form game has a NE. Once we have verified that a NE exists, we have to determine whether it is a *unique* equilibrium point. If the players have identified various Nash equilibria, it still might be difficult for them to coordinate on which one to choose. For example, in the *Multiple Access Game* both players know that there exist two Nash equilibria, but each of them tries to be the winner by deciding to transmit. Hence, their actions will result in a profile that is not a NE.

Pareto-optimality and Price of Anarchy

One method to assess the efficiency of the equilibrium point in a game is to compare the strategy profiles using the concept of Pareto-optimality. To introduce this concept, we first define Pareto-superiority.

***Definition*:** The strategy profile s is *Pareto-superior* to the strategy profile $s^{'}$ if for any player i:

$$u_i(s_i, s_{-i}) \geq u_i(s_i^{'}, s_{-i}^{'}) \tag{1.3}$$

with strict inequality for at least one player. In other words, the strategy profile s is *Pareto-superior* to the strategy profile $s^{'}$, if the payoff of a player i can be increased by changing from $s^{'}$ to s without decreasing the payoff of other players. Note that the players might need to change their strategies simultaneously to reach the Pareto-superior strategy profile s. Based on the concept of Pareto-superiority, we can identify the most efficient strategy profile or profiles.

***Definition*:** The strategy profile s^{po} is *Pareto-optimal* if there exists no other strategy profile $s^{'}$ that is Pareto-superior to s^{po}.

In a Pareto-optimal strategy profile, one cannot increase the payoff of player i without decreasing the payoff of at least one other player. Thereby, using the concept of Pareto-optimality, we can eliminate poor Nash equilibria by selecting those with a Pareto-superior strategy profile. In the *Multiple Access Game*, both pure strategy profiles (T, Q) and (Q, T) are NE and Pareto-optimal. Finally, a metric to measure the quality of a given NE is the *Price of Anarchy* (PoA). First defined in [21], the PoA is the ratio between the worst NE of the game and the Pareto-optimal.

1.2.2 Auction Design

In economics, an auction is a method to determine the value of a commodity that has an undetermined or variable price. In progressive auctions for example, the players propose increasing bids for a good, and the highest bid wins the auction. For the first time in July 1994, the FCC allocated the commercial spectrum via competitive auctions instead of the previous *best public use* method. Auctions are a suitable way to assign the spectrum licenses because it is the player who values the most the spectrum who obtains it. Still, the process can be long and can lead to an overestimated price due to strong competition (e.g., UMTS spectrum). Hence, the rules for designing and conducting spectrum auctions has evolved towards more efficient mechanisms inspired by principles of game theory. Vickrey introduced a type of sealed-bid auction in which the highest bid wins, but the price paid is the second highest bid. This mechanism provides an incentive to bidders to declare their true evaluation to maximize their payoff. An auction is called *efficient* if it maximizes the total payoffs of all players (bidders). When multiple divisible goods are sold individually, a generalization of the Vickrey auctions [22] maintains the incentive to bid truthfully, the *Vickrey-Clarke-Groves* (VCG) auctions.

The generalized VCG mechanism is an efficient auction scheme achieving the socially optimal (i.e., maximum of the total payoff; which is also a Pareto optimal, but the reverse is not true) allocation for players with quasi-linear payoff functions. The idea is that the bidders pay the opportunity cost that their presence introduces to all the other players. In a generalized VCG auction, players report their payoff to the auctioneer. The auctioneer then computes the optimal allocation that maximizes the aggregated payoff and allocates the resource accordingly. The auctioneer also solves other optimization problems: How to calculate, for each bidder, the price that each bidder should pay. The VCG auction is *truthful*, which means that each players reports her true valuation independently of the report of the other players. Each player only needs to know his own payoff function, and then the social optimal solution can be found in one-iteration.

Despite their advantages, VCG auctions have several shortcomings when applied to spectrum sharing. VCG auctions generate a large communication overhead and require high computational resources for the auctioneer to calculate the optimal payments. Hence, in Section 1.5.2, we will discuss an alternate auction scheme that is simpler to implement in wireless networks.

1.2.3 Graph Coloring

Graph coloring consists in assigning a color to the vertices of a bidirectional graph $G = (V, E)$, where V is a set of vertices and E a set of edges. The coloring is a mark that defines the category of the vertex. Marking each vertex of a graph with a finite set of k colors is equivalent to partitioning the vertices into k categories. In the following, graph coloring will refer to the coloring of the vertices of a graph. A coloring that uses at most k colors is called a *k-coloring*. Channel allocation problems, for example, can be solved by graph coloring algorithms. Let us assume that interferences in a wireless network

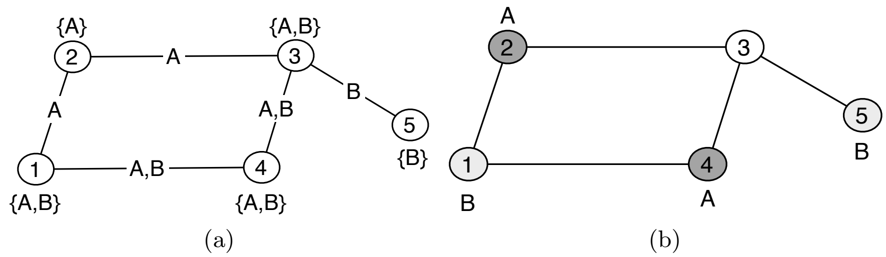


Fig. 1.1. Example of graph coloring algorithm: (a) Conflict Graph representation of a wireless network with two channels A and B. The edges are labeled to indicate interfering channels. (b) Resulting proper 2-Coloring of the Conflict Graph. Note that node 3 could not be assigned a color.

with two channels are modeled as a conflict graph where each vertex is a mobile user and nodes are connected if they interfere as shown in Fig. 1.1(a). The conflict graph can be reduced to a colored conflict graph representation in which colors map to channels. To minimize interference, the same channel must not be assigned to adjacent nodes and the problem is solved by finding a *proper k-coloring* of the conflict graph. A coloring is *proper* if no two adjacent vertices are assigned the same color (e.g., Fig. 1.1(b)). Traditional graph coloring algorithms minimize the number of colors used to mark each vertex. The best strategy consists in coloring the most difficult vertices first. As graph coloring algorithms are NP-Complete, optimal graph coloring solutions are obtained via approximations [11]. In graph coloring approximation, the idea is to prioritize the graph coloring process, instead of exhaustively testing all color assignments. *Labels* are computable values satisfying a number of predefined conditions known as *rules*. A label represents the importance of a vertex in the coloring process.

A simple sequential heuristic solution of a graph coloring problem assigns to each vertex a unique label and colors the vertices starting from the highest label with the lowest indexed color, while not violating the constraints (e.g., interference). The algorithm then removes the colored vertex and the associated edges from the graph and repeats the procedure until all vertices are colored. The prioritization of coloring via labels is a good approximation for solving graph coloring problems and it offers flexibility to solve the problem. Indeed, by varying the *rules* definitions, the algorithm can be used to minimize the number of colors for example, or, maximize the fairness of the graph coloring.

1.3 Network Operator Games

In this section, we address the game models that are proposed to design the spectrum sharing mechanisms for network operators with a part of the licensed band spectrum. Section 1.3.1 presents an asymmetric game that considers both licensed and unlicensed bands in order to design efficient networks by a single operator. In the games presented in Section 1.3.2 and 1.3.3, the network operators control the transmission power of the pilot signal at their Base Stations (BSs). Their objective is to attract as many users as possible to their BSs.

1.3.1 WAN-WiFi Competition

Zemlianov and De Veciana [30] study the problem of competition or cooperation in a multi-operator network where WAN BSs operating in a licensed band and WiFi hotspots in an unlicensed band coexist. They investigate how the decision mechanisms of the mobile users affect the ability of networking entities in such heterogeneous networks. In their scenario, the users can choose to connect to a wireless WAN with full coverage or a WiFi with limited coverage if available in their location. They use a stochastic geometric model that allow them to model channel diversity, the interference, and the spatial load fluctuations of the wireless environment.

System Model

Zemlianov and De Veciana define three point processes Π^a, Π^h, and Π^w in order to represent the location of mobile users, WiFi hotspots, and WAN BSs respectively. They suppose that the wireless WAN operator can cover the whole spatial location whereas the WiFi operator has limited coverage. Two strategies are considered for the mobile users to choose between the WAN and WiFi. With the first strategy, called *proximity-based*, the users connect to the nearest service provider (i.e., WAN BSs or WiFi AP) whereas in the second one, called *utility-based*, they can select the operator based on their payoff function.

The authors assume that each mobile makes a decision to choose its operator periodically. In the utility-based mechanism, a mobile user switches to WAN BS w_m from hotspot h_k if and only if she was connected to h_k at t^- and $u_i^w(N_m^w(t^-)+1) > u_i^h(N_k^h(t^-)+c^w)$, where u_i^w and u_i^h are the payoffs of user i when it is served by BS w_m and hotspot h_k respectively, N_k^h is the number of users connected to h_k, N_m^w is the number of user connected to w_m, t^- refers to the time immediately before t, and c^w is the cost of switching to WAN BS. A similar condition can be written for a mobile user who wants to switch from WiFi to WAN operator.

Results

Zemlianov and De Veciana prove that if the payoff function of mobile user is a continuous and monotonically decreasing function of N_k^h or N_m^w, given any initial configuration of the agents' choices at time t, the system converges to an equilibrium configuration at $t \to \infty$. This equilibrium configuration is obtained by constructing a feasible path for the chain evolution that hits an equilibrium state with positive probability, starting from any initial configuration. Note that this equilibrium might not be unique. But, by appropriately selecting the payoff function, the set of equilibria could be made tight. They also show that the class of payoff functions that are congestion dependent, provide much better performance to users on average than the simple proximity-based decision strategy. The above results can help the operators, with both wireless WAN infrastructure (e.g., WiMAX) and a set of WiFi hotspots, to design an efficient network. They show that by making use of the proposed optimal joint design with utility-based strategy at the mobile users, the operator can achieve a target performance and significantly reduce the resource costs in the same time.

1.3.2 National Border Spectrum Sharing

Félegyházi *et al.* [7] study a spectrum sharing game between two cellular operators along the national border of two countries. They consider the problem of strategic behavior in CDMA networks. Spectrum licenses are allocated within each country, thus leading to possible conflicts along national borders. There exist many examples where cities reside close to a national border (e.g., Geneva in Switzerland).

Game Model

Spectrum sharing on the border is modeled as a two-player non-cooperative power control game. The players are two cellular network operators (e.g., A and B) with one BS each, which provide wireless access in the same frequency band on the two sides of a national border. Hence, they share the spectrum and cause interference to each other. The *strategies* of the operators determine the pilot transmission power of their base stations. Player i's strategy is $s_i = P_i$, where $0W < P_i < 10W$ is the pilot signal power of BS i. The standard value of the UMTS pilot signal is $P^s = 2W$.

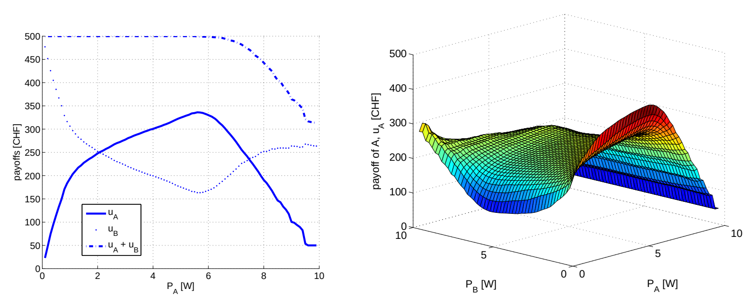
There is a set of *users* $\mathcal{M}$ equipped with *wireless devices* that access the communication network. These users select the operator with the highest pilot signal quality or *carrier-to-interference ratio (CIR)* [27] and pay for the service to the selected operator. The *CIR* is a function of pilot and traffic signal powers, the distance between user and BS, own-cell and the other-cell interferences, and the processing gain for the pilot and traffic signal from BS to user. If $\mathcal{M}_i$ represents the set of users connected to BS i and θ_u is the *expected income* obtained by serving user u of a certain traffic type, then the *payoff* function of operator i can be calculated by:

$$u_i = \sum_{u \in \mathcal{M}_i} \theta_u \tag{1.4}$$

The operators then define their strategies in order to maximize this payoff function. The main goal of this study [7] is two-fold: to establish whether the operators have an incentive to be strategic and to characterize the Nash equilibria and the Pareto-optimal strategy profiles in the game.

Game Results

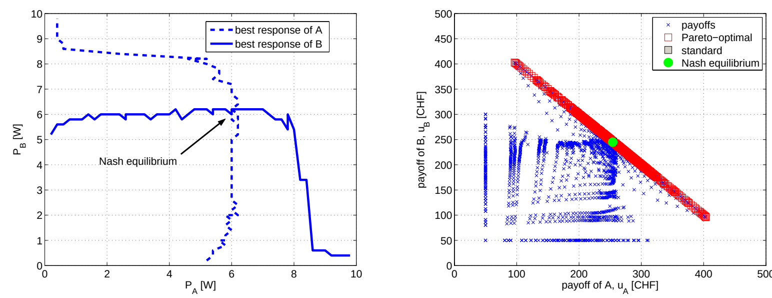
In [7], Félegyházi *et al.*, present a numerical simulation study to evaluate the above game in UMTS system using MATLAB. They distribute the mobile users according to the uniform distribution and calculate the number of users that attach to each of the BSs based on the CIR requirements over several simulations. This defines an *experimental payoff matrix* for the two players. Fig. 1.2(a) shows the payoffs of players A and B, as well as the sum of their payoffs as a function of the pilot signal power P_A. There are, on the average, 10 data traffic users in the simulation area who pay 50 CHF/month for a data traffic service. Player A is strategic by adjusting its pilot signal power, whereas player B operates his BS according to the standard pilot power P^s. The payoff function of operator A has a unique maximum point that requires a higher pilot power than P^s. The increase in payoff of player A means the decrease of the payoff of the non-strategic player B. This shows that the operator A has an incentive to be strategic. Fig. 1.2(b) shows the payoff surface for operator A as a function of the pilot power values of the two operators (i.e., when two operators are strategic). One interesting observation from this figure is that u_A has a unique maximum point for P_A. Moreover, this maximum point depends on the pilot power of the other BS, P_B.



(a) (b)

Fig. 1.2. Payoff of player A as a function of his pilot power when (a) only player A is strategic and (b) when both operators are strategic. From [7], © IEEE, 2007.

Using the two payoff surfaces, one can derive the best response functions for the operators as shown in Fig. 1.3(a). Based on the concept of best responses introduced in Section 1.2.1, the NE in the power control game can be identified. This NE point is unique for any user density and the NE pilot powers decrease as the number of users increases. The reason is that the BSs can serve enough users by using a relatively small power and hence there is no incentive for them to go above these pilot power values. Fig. 1.3(b) shows the achieved payoffs as a function of the pilot power values P_A and P_B. We observe that in this case the Pareto boundary defines a straight line, because in a Pareto-optimal strategy profile each user in the system is attached to one of the BSs. Furthermore, the standard pilot powers and the NE strategy profile result in the same payoffs for the players, and they both lie on the Pareto boundary. This means that the players achieve a desirable state, from the system point of view.



(a) (b)

Fig. 1.3. Best response and NE for border games: (a) Best response functions for the two players with 10 data users. (b) The payoff region with all possible payoffs for 10 data users. The NE, the payoff of the standard powers and all Pareto-optimal points are highlighted. From [7], © IEEE, 2007.

It is shown that the NE pilot powers are higher than the standard value. In [7], the authors extend the payoff function to include the cost of a high pilot power. The extended payoff can be defined as $\hat{u}_i = (\sum_{u \in \mathcal{M}_i} \theta_u) - C^*$, where C^* is the power cost. Hence, the players have the choice between the standard (P^s) and the NE strategies. Félegyházi *et al.* highlight the connection between this extended game and the well-known Prisoner's Dilemma.

1.3.3 Network Operators Spectrum Sharing

Félegyházi and Hubaux [8] study the spectrum sharing problem in a scenario where the mobile users can *freely roam* across the base stations (BS) of different network operators, attaching to the one offering the most favorable signal quality (i.e., the one with the strongest pilot signal) and bandwidth. This free roaming could be beneficial for both operators and users, because the former could serve an increased set of users, and the latter could enjoy various services across several operators.

Game model

The above problem is modeled as a two-player, nonzero-sum game. The players are two cellular network operators. The strategy of the operators is to define the radio range of their BSs (which is determined by their pilot signal strength). Félegyházi and Hubaux assume that the players apply the same radio range for each of their BSs. Accordingly it is assumed that they use the following radio ranges: r_H (i.e., heavy player with larger radio range) and r_L (i.e., light player with smaller radio range). The authors consider a scenario where BSs are symmetrically placed on a grid with a minimum distance d. this means that each BS of a given operator has four neighboring BSs that belong to the other operator. Consequently, the game can be analyzed considering two neighboring base stations, as shown in Fig. 1.4. The *useful coverage area* *(O_i)* for any BS i is its Voronoi calculated from the radio ranges of BS i and the radio ranges of its neighbors. The *interference area (Y_i)* for a BS i (i.e., the area, which is in its radio range, but it does not cover eventually) can be expressed as:

$$Y_i = r_i^2 \cdot \pi - O_i \tag{1.5}$$

where r_i denotes the radio range of BS i (i.e., the player i's strategy). The players want to maximize the area they cover with their pilot signals, while minimizing their interference area. Hence, the authors express the payoff function of player i (i.e., the payoff of her BS) as follows:

$$u_i(r_i, r_j) = O_i - \gamma_i \cdot Y_i = (1 + \gamma_i) \cdot O_i - \gamma_i \cdot r_i^2 \cdot \pi \tag{1.6}$$

where $\gamma_i \geq 0$ is a *cooperation parameter* that defines how much player i cares about the size of its interference area. Two cases can be distinguished for the coverage range of heavy and light player. In the first case, both players have a non-empty coverage area as presented in Fig. 1.4(a). In the second case, the light player is overwhelmed by the heavy player, meaning that the pilot signal of the heavy player is the strongest everywhere, as shown in Fig. 1.4(b). The authors assume that there exists a maximum transmission power P_{max} defined by the regulator of the wireless spectrum, which determines the maximum radio range R_{max}. They also assume that the operators want to cover the total service area. If the radio ranges of all base stations are equal, the minimum radio range for which there is full coverage can be calculated by $R_{min} = \frac{\sqrt{2}}{2}d$. Considering the limit case, in which the operators cover only the service area, one can write the following bounds on r_L and r_H:

$$\sqrt{d^2 - \sqrt{2}dr_L + r_L^2} \leq r_H \leq R_{max} \tag{1.7}$$

$$\max\{0, \frac{\sqrt{2}}{2}(d - \sqrt{-d^2 + 2r_H^2})\} \leq r_L \leq r_H \tag{1.8}$$

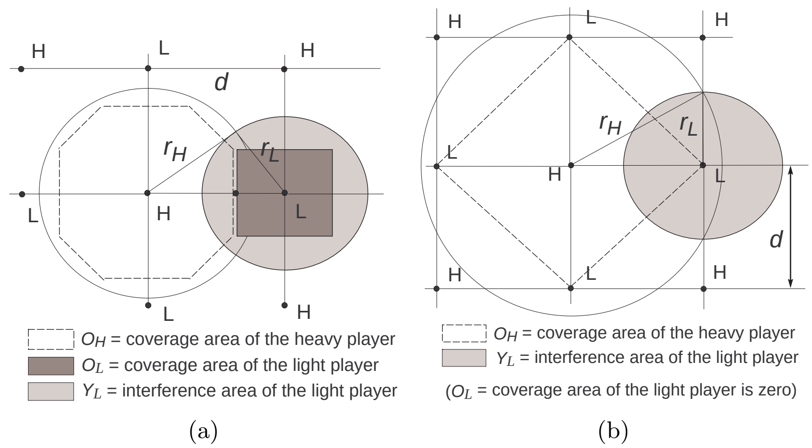


Fig. 1.4. Coverage and interference area of a base station, illustrated with two base stations: (a) both BSs have a coverage area; (b) the BSs of the light player are overwhelmed by the BSs of the heavy player and thus the light player has no coverage area at all. From [8], © IEEE, 2006.

Game Results

Félegyházi and Hubaux first study the results of a *single-stage* game (i.e., both players simultaneously choose their radio range once and for all.). They show that there exist a variety of Nash equilibria depending on cooperation parameters (i.e., γ_i and γ_j) and maximum radio range (see Table III in [8]). They also identify the Pareto optimal solutions as:

- If the operators are cooperative, they should play the radio range with which they are able to cover the service area (i.e., the lower limit in Equation (1.7)).
- If one of the players does not cooperate and the other does, then the non-cooperative player can increase its radio range to force the cooperative player out of the game (i.e., $r_i = d$, $r_j = 0$).
- If neither of the players cooperates, then they both will end up in playing the maximum radio range (i.e., $r_i = r_j = R_{max}$).
- In a fair solution, they should both play the minimum radio range (i.e., $r_i = r_j = R_{min}$).

The authors also prove a condition for which the socially desirable NE (i.e., $r_i = r_j = R_{min}$) exists and that it can be enforced using punishments in a *repeated game*. In the proposed repeated game, if operator i uses the Punisher strategy, it plays R_{min} in the first step. For any further time step, it plays: *(i)* R_{min} in the next time step if the other player played R_{min} in the previous time step; or *(ii)* R_{max} for the next k_i time steps, if the other player played anything else.

The parameter k_i (also called the *punishment interval*) defines the number of time steps for which player i punishes the other player. The Punisher strategy is similar to the well-known *Tit-For-Tat (TFT)* strategy [2], but it retaliates any defection by playing R_{max} instead of copying the same behavior. It is also different from the Trigger strategy defined in [22] (or infinite punishment), because the Punisher strategy imposes a punishment that is comparable to the amount of misbehavior and thus it is able to recover from erroneous defections. Fig. 1.5 illustrates the average per time slot payoff of a player for both cooperation and defection. One can observe that cooperation is more beneficial, because defection is quickly retaliated by the other player. Finally, the authors show that the solution of the power control problem is NP-complete for a general topology of base stations.

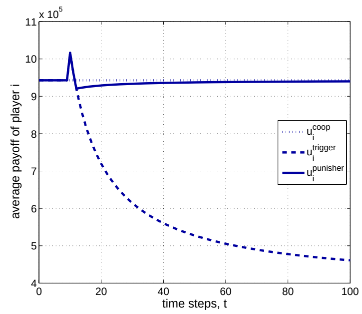


Fig. 1.5. Average payoff of player i for $d = 1km$, $\gamma_j = 0.1$ and $R_{max} = 1.5km$, if player j applies a punishment. One-time defection is quickly retaliated and hence cooperation is the best choice. The Trigger strategy stabilizes in infinite punishment, and the Punisher strategy returns to the cooperative state. From [8], © IEEE, 2006.

1.4 Games in Unlicensed Bands

As the name suggests, the unlicensed bands are the radio spectrum that can be freely used without obtaining a license. In 1986, the FCC provisioned for the first time unlicensed bands for Industry, Science, and Medicine (ISM) applications based on spread-spectrum technologies in the 915 MHz, 2.4 and 5.7 GHz spectrum bands. In the 90s, the FCC allocated additional unlicensed bands at 2, 5, and 59-64 GHz for wireless applications which require small coverage. Spectrum sharing in unlicensed bands suffers from two main problems: *(i)* devices accessing unlicensed band may experience severe interference as they do not have exclusive access to the spectrum, or worse, *(ii)* spectrum sharing in unlicensed bands may result in the *tragedy of the commons* [15] as there are no inherent incentives to efficiently use the radio band [19]. In the following we study these two problems in the context of spectrum sharing among heterogeneous wireless systems [5] and spectrum sharing among WiFi operators [13].

1.4.1 Spectrum Sharing among Heterogeneous Wireless Systems

We first consider the situation where heterogeneous wireless systems (e.g., Bluetooth and IEEE 802.11 WiFi) share the spectrum of an unlicensed band where each system behaves selfishly and tries to maximize its transmission rate. Etkin *et al.* model and study in [5] the resulting interaction among the systems as a non-cooperative game, and proposed various spectrum sharing rules and protocols to allow the wireless devices to share the bandwidth in a fair and efficient way.

One-Shot Game Model

Suppose that M wireless systems, each consisting of a single transmitter-receiver pair, share an unlicensed band of W Hz. Let $p_i(f)$, $f \in [0, W]$, be the power spectral density of the transmitted signal in system i, $i = 1, \ldots, M$, where the total power for each system can not exceed P_i, i.e.,

$$\int_0^W p_i(f) df \leq P_i. \tag{1.9}$$

Each system decides on a power allocation in order to maximize its transmission rate as follows. Given the power allocations of all other systems, system i chooses a spectral density $p_i(f)$, $f \in [0, W]$, that maximizes

$$R_i = \int_0^W log \left(1 + \frac{|h_{i,i}|^2 p_i(f)}{n_0 + \sum_{i \neq j} |h_{j,i}|^2 p_j(f)} \right) df \tag{1.10}$$

where $h_{j,i}$ is the channel gain between the sender of system j and the receiver of system i, and n_0 is the background noise power. Assuming that each system has perfect information, i.e., each system knows all the channel gains $h_{i,j}$, $i, j = 1, \ldots, M$, and all power constraints P_i, $i = 1, \ldots, M$. Etkin *et al.* show in [5] that frequency-flat allocation given by

$$p_i(f) = \frac{P_i}{W}, \qquad i = 1, \ldots, M$$

, is always a NE for the above game. Furthermore, if $\sum_{j \neq i} \frac{|h_{j,i}|^2}{|h_{i,i}|^2} < 1$, then the frequency-flat allocation is the only NE. The rate allocation under the frequency-flat allocation is generally not Pareto efficient [5], and the outcome of the game may lead to a poor overall system performance.

Repeated Game Model

To improve system performance, Etkin *et al.* then study the above situation as a repeated game. At time step t, $t \geq 0$, each system i decides on its spectral allocation $p_i(f)$, $f \in [0, W]$ subject to the constraint that the total power can not exceed P_i. The payoff u_i of system i is then given by

$$u_i = (1 - \delta) \sum_{t=0}^{\infty} \delta^t R_i(t) \tag{1.11}$$

where $R_i(t)$ is the maximal rate that system i can achieve at step t (see Equation (1.10)), and $\delta \in (0, 1)$ is a discount factor that accounts for the delay sensitivity of the system.

Consider the achievable rate region $\mathcal{R}$ for the above system, i.e., $\mathcal{R}$ is the set of rate vectors $(R_1, \ldots, R_M)$ for which there exist spectral density functions $p_i(f)$, $i = 1, \ldots, M$, that achieve $(R_1, \ldots, R_M)$ by using the relation given by Equation (1.10). Furthermore, consider the following strategy for system i. Let $(R_1, \ldots, R_M)$ be a rate vector in the achievable rate region $\mathcal{R}$ and let $p_i(f)$, $i = 1, \ldots, M$, be the corresponding spectral densities. *(i)* Then at $t = 0$: system i uses the above power allocation $p_i(f)$, and *(ii)* at time $t = t_0$, if at time $t = t_0 - 1$ every system j, $j = 1, \ldots, M$, uses the $p_j(f)$ then system i uses $p_i(f)$ at time $t = t_0$; otherwise system i uses the frequency-flat allocation, i.e., uses P_i/W for $f \in [0, W]$.

Etkin *et al.* show in [5] that for every rate vector $(R_1, \ldots, R_M) \in \mathcal{R}$ there exists a threshold $\delta_0 < 1$ such that if the discount factor δ is larger than δ_0, then the above strategy is a NE for the repeated game. Under the NE, the systems will cooperate as long as no systems deviates from $(p_1(f), \ldots, p_M(f))$ corresponding to the rate vector $(R_1, \ldots, R_M)$. A deviation by a system, will trigger a "punishment" where all systems adapt the frequency-flat power allocation leading to a poor NE. In particular, the above strategy can be used to select a Pareto efficient power allocation as the NE.

1.4.2 Spectrum Sharing among WiFi Operators

Next, we consider the situation where several WiFi operators share a common unlicensed band that is sub-divided into a fixed number of channels. Each WiFi operators owns several Access Points (AP) and has to decide on the channel that each AP uses. If two APs (of the same or two different WiFi operators) are within a (sufficiently) small distance of each other, then they will interfere if both are assigned the same channel. Therefore, in order to ensure an acceptable level of service to their mobile users, neighboring APs must be assigned different channels. Halldorsson *et al.* [13] model the above channel assignment problem as a game between WiFi operators where each operator decides on a channel assignment for its own APs in order to maximize the total number of mobile users that it can serve. The outcome of the game is evaluated by means of the Price of Anarchy (PoA) (see Section 1.2.1). The PoA measures how far the outcome of the game is from a social optimal channel allocation (i.e., a channel allocation that maximizes the total number of mobile users that are served by APs).

Game Model

Consider a set V of APs that are owned by several WiFi operators. Let $d(u, v)$, $u, v \in V$, be the distance between the two APs u and v, and let $R_t(u)$ and $R_s(u)$ be the transmission and sensing range of AP u. Note that $R_t(u)$ and $R_s(u)$ depend on the transmission power of AP u. Let $G = (V, E)$ be the corresponding interference graph where there is an edge between AP u and v if they are located close enough (i.e., if $d(u, v) \leq R_t(u)+R_t(v)+\max\{R_s(u), R_s(v)\}$). We say that AP u and v are neighbours if there is an edge in G between u and v.

There are k channels available in the unlicensed band. Operators set up (activate) APs sequentially where the order with which APs are set up is given by an exogenous process (i.e., operators do not decide when to activate an AP). Whenever an AP is set up, the corresponding operator must choose a channel that does not interfere with any of the previously set up APs. In addition, an operator is allowed to change the channel assignments of the APs that it controls as long as it does not cause any interference with APs of other operators. When an operators sets up an AP, it only has knowledge about the channel used by neighbouring APs that have already been set up. It does not have any knowledge about the channels used by previously set up APs that are outside the interference region of the AP nor does it have any knowledge about the order with which APs are set up. The payoff that an operator receives for setting up an AP is equal to the expected number of mobile users that it can serve with the AP, where different APs can have different payoffs. The goal of each operator is to choose channels in order to maximize the overall payoff of its APs.

Channel Allocation Results

For the above game, each NE corresponds to a maximal k-colored subset of the graph G. This allows to use graph theory results to compute the price of anarchy for the above game. In particular, Halldorsson *et al.* derive in [13] the following results. For the general case where APs can have different transmission powers and different payoffs, the price of anarchy is potentially unbounded, i.e., $PoA = \infty$ (see Fig. 1.6). The price of anarchy is also unbounded for the case where APs have different transmission powers, but all APs have the same payoff. If all APs have the same transmission power (i.e., the interference graph is a unit disk graph) and the same payoff, then the price of anarchy is at most $5 + \max(0, 1 - 5/k)$ and at least 5.

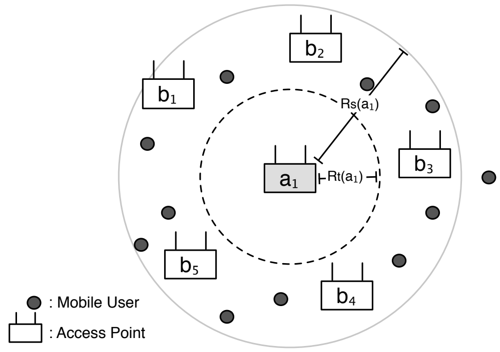


Fig. 1.6. Network operators A and B with APs $a_i, b_i \in V$ provide the Internet access to mobile users. The mobile users do not have Internet access because they are in the proximity of operator B, while operator A controls the channel. The PoA increases with the number of mobile users and is potentially infinite.

Local Bargaining

The above results show that if operators are forced to decide on a channel as soon as it is set up and are only allowed to reassign channels among the APs they control, the above game can lead to a poor coverage. An approach to improve performance is to allow operators to negotiate changes to the channel assignments of the APs that they control. Halldorsson *et al.* consider in [13] such an approach where operators can use channel bargaining to locally optimize their total payoff. In particular, they consider two bargaining schemes called *local 2-buyer-1-seller bargains* and *local 1-buyer-multiple-seller bargains*. Fig. 1.7 illustrates the local 2-buyer-1-seller bargain.

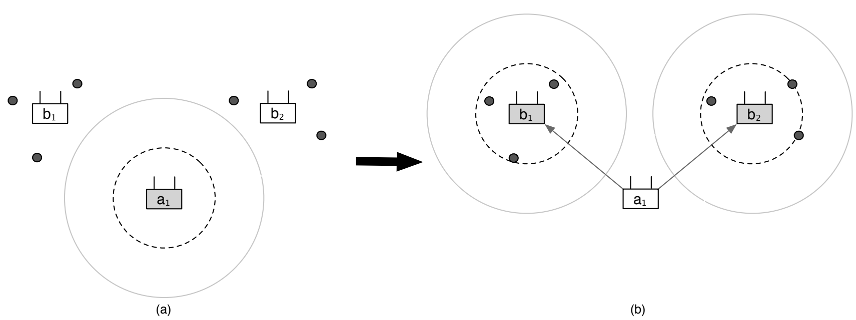


Fig. 1.7. Example of 2-buyer-1-seller bargain: (a) a_1 controls the channel (i.e., is colored), but the sum of b_1 and b_2 payoffs (i.e., the number of mobile users) is greater than a_1 payoff. (b) b_1 and b_2 bargain with a_1 to acquire the channel and improve the system payoff.

Game Results

Halldorsson *et al.* show in [13] that for the general case where APs have different transmission power and different payoffs, the price of anarchy is unbounded even if local 2-buyer-1-seller bargains or local 1-buyer-multiple-seller bargains are allowed. For the case where all APs use the same transmission power and have the same payoff, the price of anarchy under local 2-buyer-1-seller bargains is at most $3 + \max(0, 1 - 3/k)$ and at least 3. For local 1-buyer-multiple-seller bargains the price of anarchy is at most $5 + \max(0, 1 - 5/k)$ and at least 5 for the case where all APs use the same transmission power but have different payoffs. Halldorsson *et al.* also show that the above bargaining schemes will converge to a NE after a polynomial number of steps as a function of the number of APs, given that the payoffs are integers bounded by a polynomial in the number of APs. Halldorsson *et al.* also consider in [13] more general bargaining schemes than local 2-buyer-1-seller bargains or local 1-buyer-multiple-seller bargains. They prove that generally local bargaining may still lead to a poor performance unless the channel assignment of a large number of APs can be changed (i.e., global bargaining) at each bargaining step.

1.5 Cognitive Radio Games

Cognitive radios can detect whether a certain radio band or channel is currently used, as well as sense the amount of interference (interference temperature) within a given radio band or channel [16]. In addition, they are able to control the transmission powers and dynamic spectrum management with the help of *software defined radios* [16,20]. These capabilities open the possibility of a flexible sharing of the wireless spectrum [20].

In this section, we focus on the scenarios where cognitive radios are used to efficiently share the available spectrum. We first consider the situation where a primary user (operator) acquires and owns a licensed radio band. If the primary user does not fully utilize this band, then it can be accessed by secondary users (cognitive radios), as long as they do not create any (or a sufficiently small) interference to the primary user. In particular, we consider the the following two cases: (1) secondary users can freely utilize the radio spectrum as long as they do not interfere with the primary user (Section 1.5.1) and (2) the primary user sells access to the radio band through an auction mechanism (Section 1.5.2). In Section 1.5.3, we consider an OFDM network where several cognitive radios share the available OFDMA channels.

1.5.1 Opportunistic Spectrum Sharing

We first consider the situation where several primary users (operators) acquire their own radio band and where each radio band is further divided into several channels. We focus on an opportunistic spectrum sharing where secondary users (cognitive radios) are free to utilize channels as long as they do not interfere with the primary users [4, 26, 32]. Here, it assumed that secondary users will cooperate with each other to obtain a channel allocation with a maximum utilization subject to a given fairness criteria. Fig. 1.8 provides an example of opportunistic spectrum sharing where the unused spectrum from a TV broadcast channel is utilized to provide WiFi connections to a residential community. Secondary user 2 cannot make use of channel A as it would interfere with primary user X. Secondary users 1 and 3 can emit on channel A as long as they control their transmit power not to interfere further than $d_s(1, A)$ or $d_s(3, A)$ respectively.

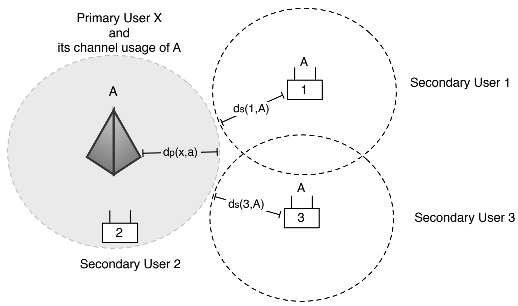


Fig. 1.8. Secondary users 1 and 3 exploit channel A with their WiFi APs without interfering with the base station of the primary user.

System Model

Consider the situation where N secondary users share M channels. Different channels allow different secondary users to transmit at different rates. Let $b_{n,m}$ be the throughput that user n can achieve on channel m, where $n = 1, \ldots, N$ and $m = 1, \ldots, M$. The interference constraints are modeled by an interference graph (see for example [26]) which defines on which channels a given secondary user does not interfere with the primary user, and which secondary user can simultaneously transmit on a given channel m without causing interference among themselves. In addition, it is assumed that the maximum number of channels assigned to a secondary user can not exceed a given threshold $C_{\max}$. For a given feasible channel allocation (i.e., a channel allocation that does not violate any interference constraints and any of the constraints on the maximum number of channels allocated to a secondary user), the network utilization is defined as the sum of the throughput over all secondary users.

Results

Using the above model, Peng *et al.* formulate in [26] the channel allocation problem as a graph coloring problem where the goal is to maximize network utilization, or to maximize network utilization subject to a given fairness criteria such as max-min fairness and proportional fairness [18]. Finding such an optimal channel allocation is NP-hard and Peng *et al.* propose several heuristic graph coloring algorithms for each of the above performance objective. Peng *et al.* derive lower bounds for the performance of the centralized as well distributed implementation of these algorithms.

The algorithms proposed by Peng *et al.* in [26] require coordination and frequent information exchange among secondary users, and may impose substantial overhead on the network. As an alternative approach, Zheng and Cao in [32] propose a so-called device-centric management scheme where each secondary user accesses channels based on simple rules that require only local information. The authors propose several allocation rules and derive lower bounds for their performance in terms of the poverty line (i.e., the minimum number of channels each secondary user is guaranteed to obtain), An important property of the proposed algorithms is that they reach a stable channel allocation in a finite number of iterations.

In [4], Cao and Zheng allow secondary users to be mobile. Instead of computing the channel allocation at each network topology change, secondary users negotiate channel allocations with their neighborhood via local bargaining. For the proposed local bargaining mechanism, a theoretical bound on the lower bounds for their performance in terms of the poverty line is provided.

1.5.2 Auction Based Spectrum Sharing

Next, we consider the situation where a primary user (operator or government agency) lets secondary users access its spectrum subject to a given power constraint, i.e., the total interference created by the secondary users at fixed measurements points has to be below a given threshold. For this situation, Huang *et al.* propose in [17] an auction-based spectrum sharing where secondary users submit bids. Based on these bids, the primary user decides on the transmission power allocated to each secondary user, as well as the cost per unit transmission power that secondary users are charged. The goal of each user is to submit bids in order to maximize its payoff minus cost, where the payoff is a function of the received signal-to-interference plus noise ratio.

System Model

Consider M secondary users and let p_i be the transmission power of user i, where $i = 1, \ldots, M$. The primary user then allocates transmission power to secondary users such that the total received power at a given measurement point is less than a threshold P_{max} (i.e., $\sum_{i=1}^{M} p_i h_{i0} \leq P_{max}$, where h_{i0} is the channel gain from user i's transmitter to the measurement point). Let

$$\gamma_i = \frac{p_i h_{ii}}{n_0 + \kappa\left(\sum_{j \neq i} p_j h_{ji}\right)}$$

be signal to noise and interference ratio ($SINR$) at secondary user i's receiver, where κ is a positive constant that depends on the spectrum bandwidth, h_{ji} is the channel gain from secondary user j to secondary user i's receiver, and n_0 is the background noise power. The payoff function of secondary user i is a function of γ_i given by $u_i(\gamma_i) = \theta_i ln(\gamma_i)$, where θ_i is a user-dependent parameter.

Auction Based Allocation

For the above situation, Huang *et al.* consider the problem where the primary user wants to allocate transmission power to secondary users in order to maximize the social welfare [17] subject to the interference constraint at the measurement point. As discussed in Section 1.2.2, a VCG auction could be used to achieve this. However, a VCG auction might not be suitable for this situation due to (a) the overhead of communicating the payoff function to the primary user and (b) the computational complexity of computing an optimal allocation. As an alternative approach, Huang *et al.* propose the following auction based power allocation scheme.

The primary user decides on a reserve power p_0 and announces a reserve bid $\beta \geq 0$ and a price $\pi^s > 0$. After observing β and π^s, each secondary user i submits its bid b_i. The primary user then allocates transmission power p_i to secondary users so that the received power at the measurement point is proportional to the bids, i.e., we have

$$h_{i0} p_i = \frac{b_i P_{max}}{\sum_{j=1}^{M} b_j + \beta}, \; and \;\; p_0 = \frac{\beta P_{max}}{\sum_{j=1}^{M} b_j + \beta}, \tag{1.12}$$

and charges each secondary user a price $C_i = \pi^s \gamma_i$ where γ_i is the $SINR$ of secondary user i under the above power allocation. The goal of each secondary user is to submit a bid in order the resulting power allocation maximizes its payoff $u_i(\gamma_i)$ minus cost C_i. Assuming complete information, Huang *et al.* model this situation as a non-cooperative game, and prove that for $\beta > 0$, there exists a threshold price $\pi_{th}^s > 0$ such that a unique NE exists if $\pi^s > \pi_{th}^s$; there does not exist a NE if $\pi^s \leq \pi_{th}^s$. For this game Pareto optimality and stability (NE) are conflicting, however an ϵ-Pareto optimal NE can be achieved. The ϵ-Pareto optimal allocation is the Pareto-optimal solution for the ϵ-system in which the total received power at measurement point is less than $(1-\epsilon)P_{max}$.

The assumption that secondary users have complete information when deciding on their bids is unrealistic, and Huang *et al.* propose an iterative bidding algorithm that requires each secondary user to have access only to his local information (i.e., its own payoff function and its local channel gains) and therefore can be implemented in a fully distributed manner. It is shown that this algorithm converges to the NE of the complete information game.

1.5.3 Spectrum Sharing in OFDM Networks

In this section we study spectrum sharing among cognitive radios in OFDMA networks [14]. OFDMA is a transmission technique which divides the available spectrum into sub-carriers and hence can support different QoS by assigning different number of sub-carriers to the users. OFDMA has recently used in the WiMAX (IEEE 802.16 Wireless MAN) uplink. In the following, we consider the situation where several cognitive radios, each having its own QoS constraint in terms of throughput, compete for access to the available sub-channels in an OFDMA network. Note that, there is not any spectrum manager (auctioneer) in this OFDMA network.

System Model

Consider an OFDMA network consisting of L sub-channels that are shared among K users (cognitive radios). Each user has a given QoS constraint in terms of throughput, (i.e., let R_i, $i = 1, \dots, K$, be the required transmission rate of user i). Consider a given power allocation given by the matrix P where $[P]_{il} = P_i^l$ is the transmission power of user i on sub-channel l, $l = 1, \dots, L$. For a given power allocation P, Let

$$\gamma_i^l(P) = \frac{P_i^l h_{ii}^l}{n_0 + \sum_{j \neq i} P_j^l h_{ji}^l}, \qquad l = 1, \dots, L, \tag{1.13}$$

be $SINR$ for user i on sub-channel l, where h_{ji}^l is the channel gain from user j to user i's receiver (e.g., user i's BS) on sub-channel l, and n_0 is the background noise power. The rate $r_i^l(P)$ at which user i can transmit on sub-channel l under the power allocation P is then given by $r_i^l(P) = W \log_2(1 + c\gamma_i^l(P))$, where W and c are known positive constants that depend on the system parameters. The goal of an OFDMA network provider is to minimize the overall transmission power subject to the given QoS constraints, i.e.,

$$\min_P \sum_{i=1}^{K} \sum_{l=1}^{L} P_i^l \tag{1.14}$$

$$s.t. \begin{cases} \sum_{l=1}^{L} r_i^l(P) - R_i \geq 0, & i = 1, \dots, K, \\ \sum_{l=1}^{L} P_i^l - P_{max} \leq 0, & i = 1, \dots, K, \\ P_i^l \geq 0, & i = 1, \dots, K, \text{ and } l = 1, \dots, L. \end{cases}$$

where P_{max} is a constraint on the maximal power at which a user can transmit. For the above optimization problem, we denote with Ω the set of feasible sub-channel rate allocations. In other words, Ω is the set of all sub-channel rate allocations r_i^l, $i = 1, \dots, K$ and $l = 1, \dots, L$, which satisfy the QoS constraints that is $\sum_{l=1}^{L} r_i^l(P) - R_i \geq 0$ for $i = 1, \dots, K$ and for which there exists a feasible power allocation P such that, $r_i^l(P) = r_i^l$ for $i = 1, \dots, K$ and $l = 1 \dots, L$. Note that the set Ω might be empty. Necessary conditions on the QoS constraints R_i, $i = 1, \dots, K$, for Ω to be non-empty are given in [14].

Game Model

The optimization problem given by Equation (1.14) is a generalized *knapsack* problem and finding an optimal power allocation is NP-hard [14]. Rather than relying on the network operator to decide on a power allocation, suppose that each user i is free to choose its own power allocation with the goal to satisfy the rate constraint R_i with a minimal transmission power. The resulting interaction among users leads to the following non-cooperative game. Let P_{-i} be the sub-matrix indicating the power allocation over all users except user i and let $P_i = (P_i^1, \dots, P_i^L)$ be the power allocation of user i. Furthermore, let

$$r_i^l(P_i, P_{-i}) = W \log_2 \left(1 + c\gamma_i^l(P_i, P_{-i})\right)$$

be the transmission rate of user i on sub-channel l, where $\gamma_i^l(P_i, P_{-i})$ is the $SINR$ for user i on sub-channel l under the power allocation given by P_i and P_{-i} as defined above. Given an allocation P_{-i} of all other users, user i chooses a power allocation P_i to minimize its own transmission power subject to the QoS on its transmission rate by solving the following optimization problem,

$$\min_{P_i} \sum_{l=1}^{L} P_i^l \tag{1.15}$$

$$s.t. \begin{cases} \sum_{l=1}^{L} r_i^l(P_i, P_{-i}) - R_i \geq 0, \\ \sum_{l=1}^{L} P_i^l - P_{max} \leq 0, \\ P_i^l \geq 0, \qquad l = 1, \dots, L. \end{cases}$$

Game Results

For the above non-cooperative game, Han *et al.* show in [14] that there always exists a NE if the set Ω is non-empty, i.e., if there exists at least one feasible power allocation that leads to the rate allocation that satisfies the QoS constraints R_i, $i = 1, \dots, K$. However two difficulties arise in this context: (1) if the required rates R_i, $i = 1, \dots, K$, are too large, then the set Ω might be empty and no NE exists, and (2) there might exist several NE, some of them with low system and individual performances. To overcome these difficulties, Han *et al.* consider the use of a virtual referee that is implemented by the network operator. Below we briefly describe the role of the virtual referee.

Suppose that when the set Ω is empty and no NE exists, then a user i who is not able to satisfies the QoS constraint R_i without violating the power constraint will decide on a power allocation as follows. Given a power allocation P_{-i} by all other users, user i chooses a power allocation P_i that solves the following optimization problem,

$$\max_{P_i} \sum_{l=1}^{L} r_i^l(P_i, P_{-i}) \tag{1.16}$$

$$s.t. \begin{cases} \sum_{l=1}^{L} P_i^l - P_{max} \leq 0, & \\ P_i^l \geq 0, & l = 1, \ldots, L, \end{cases}$$

i.e. user i maximizes its throughput subject to the given power constraint. Han *et al.* show in [14] that there always exists a NE for the non-cooperative game based on the two optimization problems given by Equation (1.15) and (1.16), respectively. However, the game may result in an undesirable NE with low system and individual performances. To improve system performance, the network operator implements a virtual referee that selectively restricts for some users the set of sub-channels that they are allowed to access. Han *et al.* provide in [14] the rules that the virtual referee uses to limit sub-channel access, one rule being that each user must have access to at least one sub-channel. Through numerical results, Han *et al.* illustrate in [14] that the resulting spectrum sharing mechanisms can significantly outperform two benchmark mechanisms where each sub-channels is allocated to at most one user (no sub-channel sharing) and where all users access all sub-channels (complete sub-channel sharing).

1.6 Conclusion

In this chapter, we have provided a detailed description of several research contributions in the area of spectrum sharing games. More specifically, we have focused on two kinds of players: network operators and cognitive radios. The reason of this choice is that we believe that the described scenarios will be among the most relevant ones in the coming years. They provide insights on the possible consequences of greedy behavior in the either. As we have explained, most of the cases that we have described are amenable to modeling by game theory. In this way, it is possible to predict potential outcomes of the observed conflicting situations. But as we have mentioned, in order to make the problem tractable, all authors have made relatively drastic assumptions, notably in terms of information possessed by the players. Much research is still needed in this field, in particular to better capture the perception that each of the players has of the context in which it operates.

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