Non-Cooperative Multi-Radio Channel Allocation in Wireless Networks

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Abstract— **Channel allocation was extensively studied in the framework of cellular networks. But the emergence of new system concepts, such as cognitive radio systems, has brought this topic into the focus of research again. In this paper, we study in detail the problem of competitive multi-radio multi-channel allocation in wireless networks. We study the existence of Nash equilibria in a static game and we conclude that, in spite of the non-cooperative behavior of such devices, their channel allocation results in a load-balancing solution. In addition, we consider the fairness properties of the resulting channel allocations and their resistance to the possible coalitions of a subset of players. Finally, we present two algorithms that achieve a load-balancing Nash equilibrium channel allocation; each of them using a different set of available information.**

I. Introduction

Wireless networks provide a flexible and cost-efficient method for establishing communication between different parties. Each wireless network operates in a frequency band assigned by the authorities that regulate the frequency spectrum in a given country. In general, the communication medium assigned to a given network is shared among the communication devices using some *multiple access* technique.

Frequency Division Multiple Access (FDMA) is one of the widely used techniques that enables several users to share a communication medium that consists of a given frequency band [23], [24]. The basic principle of FDMA is to split up the available bandwidth to distinct sub-bands called *channels*. Assigning the radio transceivers to these channels is commonly referred to as the *channel allocation* problem.[1] Not surprisingly, an efficient channel allocation is a cornerstone of the design of wireless networks.

In this paper, we present a game-theoretic analysis of fixed channel allocation strategies of devices that use multiple radios. Using a static non-cooperative game, we analyze the scenario of a single collision domain, i.e., where each of the devices can interfere with a transmission of every other device. We derive the Nash equilibria in this game and show that they result in load balancing over the channels. Our main results show that there exist two types of Nash equilibria: in the first type, each user distributes his radios over the available channels, whereas in the second type, some users allocate multiple radios on certain channels. We also study fairness issues and the problem of coalition formation in the channel allocation problem. We show that a Nash equilibrium that resists coalitions of users is necessarily fair as well. Furthermore, we propose three algorithms to achieve the Nash equilibria. The first is a sequential algorithm that needs global coordination, the second is a distributed algorithm that needs perfect information and the third is a distributed algorithm that is based on imperfect information. We provide the proof for the convergence properties of these algorithms.

This work is a first step towards a deeper understanding of the non-cooperative behavior of such devices and is applicable in particular in the emerging field of cognitive radio systems [15].

The paper is organized as follows. In Section II, we present related work on channel allocation and channel access in wireless networks. In Section III, we introduce our system model along with a game-theoretic description of competitive channel allocation. In Section IV, we provide a comprehensive analysis of the Nash equilibria in the channel allocation game. We study fairness issues in Section V. In addition, we provide some results on coalition-proof Nash equilibria in Section VI. In Section VII, we propose two algorithms to reach the desired Nash equilibria and study their convergence properties. Finally, we conclude in Section VIII.

II. Related work

There has been a significant amount of work on channel allocation in wireless networks, notably for cellular networks. Channel allocation schemes in cellular networks can be divided into three categories: fixed channel allocation (FCA), dynamic channel allocation (DCA) and hybrid channel allocation (HCA), which combines the two former methods.

In a fixed channel allocation scheme, the same number of channels are permanently allocated to the radios at the base stations. To study fixed channel allocation, most authors used graph coloring / labelling techniques (e.g., in [25]). The

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[1] In the literature, the terms channel assignment and frequency assignment are also used for the channel allocation problem.

FCA method performs very well under a high traffic load, but it cannot adapt to changing traffic conditions or user distributions.

To overcome the inflexibility of FCA, many authors propose dynamic channel allocation (DCA) methods (e.g. as presented in [9], [26]). In contrast to FCA, there is no constant relationship between the base stations in a cell and their respective channels. All channels are available for each base station and they are assigned dynamically as new users arrive. Typically, the available channels are evaluated according to a cost function and the one with the minimum cost is used [10]. Due to its dynamic property, the DCA can adapt to changing traffic demand. Because adaptation implies some cost, it performs worse than FCA in the case of a heavy traffic load. For a comprehensive survey on the topic, we refer the reader to [16].

Due to the emergence of alternative communication technologies, channel allocation schemes are becoming a focus of research again. Mishra *et al.* [20] propose a channel allocation method for wireless local area networks (WLANs) based weighted graph coloring. Zheng and Cao [27] present a rule-based spectrum management scheme for cognitive radios.

Recently, several researchers have considered devices using multiple radios, notably in mesh networks (for a survey on mesh networks, see [2]). In the multi-radio communication context, channel allocation and access also became one of the crucial topics. Related work on multi-radio medium access includes, but is not restricted to [1], [3], [22].

In all the related work cited so far, the authors assumed that the radio devices cooperate to achieve a high system performance. But this assumption might not hold, as the users of these devices are usually selfish and they want to maximize their own performance without necessarily respecting the system objectives. Game theory provides a straightforward tool to study medium access problems in competitive wireless networks and has been applied to the CSMA/CA protocol [7], [17] and to the Aloha protocol [19]. Furthermore, a fixed channel allocation game was presented in [14] based on graph coloring. Unfortunately, their model does not apply to multi-radio devices. For cognitive radio networks, the authors of [21] propose a dynamic channel allocation scheme based on a potential game. In addition, they suggest another technique based on machine learning with different utility functions. Cao and Zheng [8] propose distributed spectrum allocation in cognitive radio networks based on local bargaining.

III. System Model and Concepts

We assume that the available frequency band is divided into orthogonal channels of the same bandwidth using the FDMA method (e.g., 8 orthogonal channels in case of the IEEE 802.11a protocol). We denote the set of available orthogonal channels by $\mathcal{C}$.

In our model, pairs of users want to communicate with each other over a single hop. We assume that each user participates in only one such communication session, hence we denote the set of such communication links by $\mathcal{N}$. Each user owns a device equipped with $k \leq |\mathcal{C}|$ radio transmitters, all having the same communication capabilities. The communication between two devices is bidirectional and they always have some packets to exchange. Due to the bidirectional links, the sender and the receiver are able to coordinate and thus to select the same channels to communicate. Accordingly, we assume that the sender controls the communication in a certain pair and we refer to him as a *selfish player*. The objective of each player is to maximize his total throughput or channel utilization. We assume that there is a finite number of players. We further assume that each device can hear the transmissions of every other device if they are using the same channel. This means that the players reside in a *single collision domain*. We make this assumption to avoid the hidden terminal problem described for example in [24]. Because the devices reside in a single collision domain, it is reasonable to assume that the channels have roughly the same expected channel characteristics.

We assume that there is a mechanism that enables the players to use multiple channels to communicate at the same time (as it is implemented in [1] for example). We denote the number of radios of player i using channel c by $k_{i,c}$ for every $c \in \mathcal{C}$. For the simplicity of presentation, let us denote the set of channels used by player i by $\mathcal{C}_i$, where $\mathcal{C}_i \subset \mathcal{C}$ and $0 \leq |\mathcal{C}_i| \leq k$. We further assume that there is no limitation on the number of radios per channel.

We formulate the multi-radio channel allocation problem as a non-cooperative game as follows. We define the *strategy* of player i as his channel allocation vector:

$$s_i = \{k_{i,1}, \ldots, k_{i,|\mathcal{C}|}\} \tag{1}$$

Hence, his strategy consists in defining the number of radios on each of the channels.[2] The strategy vectors of all players defines the strategy matrix S (i.e., the strategy profile), where the row i of the matrix corresponds to the strategy vector of player i:

$$S = \left(\begin{array}{c} s_1 \\ \ldots \\ s_{|\mathcal{N}|} \end{array} \right) \tag{2}$$

Furthermore, we denote the strategy matrix except for the strategy of player i by S_{-i}.

Figure 1 presents an example channel allocation with six available channels ($|\mathcal{C}| = 6$), four players ($|\mathcal{N}| = 4$) and each user device equipped by four radios ($k = 4$).

The total number of radios employed by player i can be written as $k_i = \sum_c k_{i,c}$. Similarly, we can obtain the number of radios using a particular channel $k_c = \sum_i k_{i,c}$. In Figure 1, each player has a radio on channel c_1, but channel c_5 is occupied only by player p_2. Player p_3 employs two radios on channel c_2 to get more bandwidth on that particular channel. Regarding the number of radios per player, we have $k_{p_1} = k_{p_2} = k_{p_3} = 4$ and $k_{p_4} = 2$, meaning that player p_4 is not using all of his radios.

We assume that the players are rational and their objective is to maximize their *payoff* in the network. We denote the payoff of player i by U_i. For simplicity, we assume that each

[2] Note that this number can be zero.

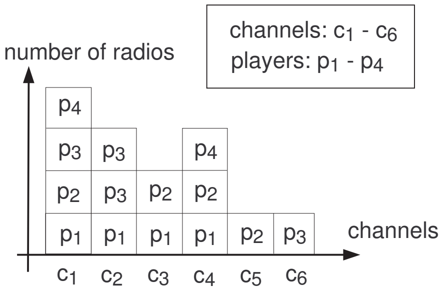

Fig. 1. An example for a channel allocation, where $|\mathcal{C}| = 6$, $|\mathcal{N}| = 4$ and $k = 4$.

player i wants to *maximize his aggregated rate* (R_i) in the system and thus the payoff function is the achieved bitrate.

We assume that the total rate on channel c is shared equally among the radio transmitters using that channel. We denote the *rate of one radio* on channel c by $R_c(k_c)$. As we assume that channels have the same bandwidth and channel characteristics, the achieved rate does not depend on the channel and thus we can write $R(k_c)$ for any channel $c \in \mathcal{C}$. The fair rate allocation is achieved, for example, by using a reservation-based TDMA schedule on a given channel. A similar result was reported by Bianchi in [6] for the CSMA/CA protocol. Even if the radio transmitters are controlled by selfish users in the CSMA/CA protocol, they can achieve this fair sharing as shown in [7]. We further assume that the *total rate* $R^t(k_c) = k_c \cdot R(k_c)$ on a channel c (i.e., the sum of the achieved throughput of all players on channel c) is a non-increasing function of the number of radios k_c deployed on this channel. In fact $R^t(k_c)$ is independent of k_c for an ideal TDMA protocol. In random access protocols, such as CSMA/CA, the total rate function $R^t(k_c)$ becomes a decreasing function of k_c for $k_c > 1$ due to packet collisions. If $k_c = 0$, we define $R^t(0) = 0$; note however that this case has no relevance in our model. We emphasize that our system model is general enough to incorporate many multiple access techniques, such as TDMA or CSMA/CA.

Figure 2 presents the total rate $R^t(k_c)$ as a function of the number of radios using channel c.

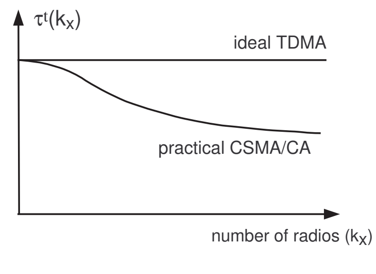

Fig. 2. The total rate $R^t(k_c)$ for different MAC protocols.

If player i chooses to operate $k_{i,c}$ radios in a given channel, his rate on this channel can be written as $R_{i,c} = k_{i,c} \cdot R(k_c)$. We assume that the players do not cheat at the MAC layer as opposed to the model for example in [7]. Thus, we can write that $R_{i,c} > 0$ for all $c \in \mathcal{C}$, where $k_{i,c} > 0$. Recall that in Figure 1, the higher the number of radios in a given channel is, the lower the rate per radio is. Hence, for example for player p_2, we have $R_{p_2,c_1} < R_{p_2,c_4} < R_{p_2,c_3} < R_{p_2,c_5}$. We can obtain the rate R_i for player i by $R_i = \sum_c R_{i,c}$.

In summary, we can write the payoff function for player i as:

$$U_i(S) = R_i = \sum_{c \in \mathcal{C}} R_{i,c} = \sum_{c \in \mathcal{C}} k_{i,c} \cdot R(k_c) \tag{3}$$

We model the channel allocation problem with a *single stage game*, which corresponds to a fixed channel allocation among the players.

In order to study the strategic interaction of the players, we first introduce the concept of Nash equilibrium [12], [13].

Definition 1: (Nash Equilibrium – NE): The strategy matrix $S^* = \{s_1^*, \ldots, s_{|\mathcal{N}|}^*\}$ defines a Nash Equilibrium (NE), if for every player i, we have:

$$U_i(s_i^*, S_{-i}^*) \geq U_i(s_i^{'}, S_{-i}^*) \tag{4}$$

for every strategy $s_i^{'}$.
In other words, in a NE none of the players can unilaterally change its strategy to increase its payoff. A NE solution is often inefficient from the system point of view. We characterize the efficiency of the solution by the concept of Pareto-optimality.

Definition 2: (Pareto-Optimality): The strategy matrix S^{po} is Pareto-optimal if $\nexists S^{'}$ such that:

$$U_i(S^{'}) \geq U_i(S^{po}), \forall i \tag{5}$$

with strict inequality for at least one player i.
This means that in a Pareto-optimal channel allocation S^{po} one cannot improve the payoff of any player i without decreasing the payoff of at least one other player j.

Finally, let us express the system inefficiency due to selfish behavior using the concept of the price of anarchy defined in [18]:

Definition 3: (Price-of-Anarchy – POA): The price of anarchy (POA) of a game is the ratio between the sum of the payoffs of all players in a globally optimal solution compared to the sum of the payoffs achieved in a worst-case Nash equilibrium.

IV. Nash Equilibria

In this section, we study the existence of Nash equilibria in the single collision domain channel allocation game. Note that we omit the proofs of the results due to space limitations, but we provide an extended version of this work in [11].

It is straightforward to see that if the total number of radios is smaller than or equal to the number of channels, then a flat channel allocation, in which the number of radios per channel does not exceed one, is a Nash equilibrium.

Fact 1: If $|\mathcal{N}| \cdot k \leq |\mathcal{C}|$, then any channel allocation, in which $k_c \leq 1, \forall c \in \mathcal{C}$ is a Pareto-optimal NE.

For the remainder of the paper, we assume that $|\mathcal{N}| \cdot k > |\mathcal{C}|$, hence the devices have a conflict during the channel allocation process.

In the following, we consider a NE strategy matrix in the multi-radio channel allocation game denoted by S^*, where

$s_i^* \in S^*$ is the NE strategy of player i (i.e., the i-th row of the matrix).

We first show the following intuitive result: a selfish player should use all of his radios in order to maximize his total rate. This is a necessary condition for a Nash equilibria.

Lemma 1: If S^* is a NE of the multi-radio channel allocation game, then $k_i = k, \forall i$.

In the example presented in Figure 1, Lemma 1 does not hold for players p_4, because it uses only two radios. Hence, the example cannot be a NE.

Let us now consider two arbitrary channels c and d. Without loss of generality, we assume that there are more radios using channel c, meaning that $k_c \geq k_d$, and denote their difference by:

$$\delta_{c,d} = k_c - k_d \tag{6}$$

Let us now divide the channels in a certain channel allocation S into three sets. We define the set of channels $\mathcal{C}^+$ with the maximum number of radios, i.e., where $c \in \mathcal{C}^+$ has $k_c = \max_{l \in \mathcal{C}} k_l$. Similarly, let us define the set of channels $\mathcal{C}^-$ for which $k_d = k_d - 1$. We denote the set of the remaining channels by $\mathcal{C}^{--}$. In Figure 1, $\mathcal{C}^+ = \{c_1\}$, $\mathcal{C}^- = \{c_2, c_4\}$ and $\mathcal{C}^{--} = \{c_3, c_5, c_6\}$.

In the following proposition, we show that in a Nash equilibrium, the difference in the total number of radios between any two channels cannot exceed one.

Proposition 1: If S^* is a NE in the multi-radio channel allocation game, then $\delta_{c,d} \leq 1$ for all $c, d \in \mathcal{C}$.

Proposition 1 shows that in a NE only $\mathcal{C}^+$ and $\mathcal{C}^-$ exist. This establishes an interesting property about NE: *In fact, all NE channel allocations achieve load-balancing over the channels in $\mathcal{C}$.* Based on Proposition 1, we express a set of sufficient conditions for the NE.

Theorem 1: Assume that $|\mathcal{N}| \cdot k > |\mathcal{C}|$. Then a channel allocation S^* is a NE if the two following conditions hold:

- ▷ $d_{c,d} \leq 1$ for any $c, d \in \mathcal{C}$ and
- ▷ $k_{i,c} \leq 1$ for any $c \in \mathcal{C}$.

An example of a NE channel allocation corresponding to Theorem 1 is shown in Figure 3.

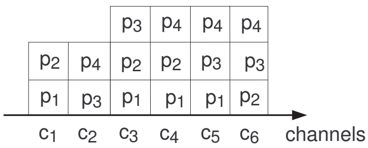

Fig. 3. A NE channel allocation corresponding to Theorem 1. Here $|\mathcal{C}| = 6$, $|\mathcal{N}| = 4$ and $k = 4$. Each player distributes his radios over the channels (i.e., $k_{i,c} \leq 1, \forall i, \forall c$).

Theorem 1 suggests that players should distribute their radios over the set of available channels. Surprisingly, there exist another type of Nash equilibria in which some players use multiple radios in some channels. We characterize these Nash equilibria in the following theorem.

Theorem 2: Assume that $|\mathcal{N}| \cdot k > |\mathcal{C}|$. Then a channel allocation S^* is a NE if the following conditions hold:

- ▷ $d_{c,d} \leq 1$ for any $c, d \in \mathcal{C}$ and
- ▷ for any player i that has $k_{i,c} \geq 2$, $k_{i,c} \leq \frac{\tau(k_c-1)-\tau(k_c+1)}{\tau(k_c-1)-\tau(k_c)}$ also holds; and
- ▷ for any player i that has $k_{i,c} \geq 2$ and $c \in \mathcal{C}^+$, it is also true that $k_{i,d} \geq k_{i,c} - 1, \forall d \in \mathcal{C}^-$

Figure 4 presents an example for Theorem 2 assuming that the second (numerical) condition of the theorem holds.

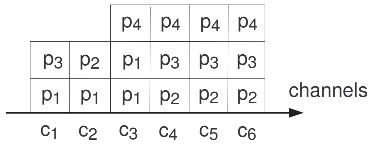

Fig. 4. An example for a NE channel allocation corresponding to Theorem 2. Here $|\mathcal{C}| = 6$, $|\mathcal{N}| = 4$ and $k = 4$. Note that player p_1 uses multiple radios on channel c_1.

In summary, Theorems 1 and 2 characterize two types of Nash equilibria. In the first type, each player distributes his radios such that he has at most one radio per channel. Intuitively, this results in load balancing. Note, however, the existence of a second type of Nash equilibria, in which some players allocate multiple radios on certain channels. We mention that there could be a small set of other Nash equilibria that are not covered by these theorems, but they exist for very specific conditions on the total throughput function $R^t(k_c)$.

In the next theorem, we will show that allowing selfish channel allocation results in an efficient spectrum utilization if the rate function is independent of the number of radios on a certain channel.

Theorem 3: Assume that $|\mathcal{N}| \cdot k > |\mathcal{C}|$ and the total rate function $R^t(k_c)$ is independent of k_c on any channel c. Then any NE channel allocation S^* is Pareto-optimal.

It is very intuitive to also state the following fact.

Fact 2: If $|\mathcal{N}| \cdot k > |\mathcal{C}|$ and $R^t(k_c)$ is a decreasing function of k_c, then every Pareto-optimal channel allocation has the property $k_c = 1, \forall c \in \mathcal{C}$.

Note that this result does not hold for decreasing total rate functions, because the players might remove some of their radios to decrease the total number of radios on certain channels. If they do this mutually, they could mutually increase each others payoff.

Using the concept of the *price of anarchy (POA)* introduced before, we can express the inefficiency caused by selfish behavior.

Theorem 4: If $R^t(k_c)$ is a decreasing function of k_c, then the price of anarchy (POA) is given by:

$$POA = \frac{R^t(1)}{(k_c + 1 - \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|}) \cdot (R^t(k_c) - R^t(k_c+1)) + R^t(k_c+1)} \tag{7}$$

where $k_c = \left\lfloor \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|} \right\rfloor$ (i.e., $k_c + 1 = \left\lceil \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|} \right\rceil$).

We can notice that if $R^t(k_c)$ is close to a constant function, then the price of anarchy is close to one. This means that for these type of total rate functions, the Nash equilibria are system-efficient.

V. Fairness issues

In this section, we study the *fairness* properties of the selfish multi-radio channel allocation game. Fairness is an important aspect of resource allocation problems in general, and of computer networks in particular. Very often, a system-efficient resource allocation gives more (or all) resources to a few players while neglecting other players (e.g. in the NE derived for the CSMA/CA protocol in [7]). Such a solution might be system-efficient, but it is not desired from the network designer's point of view (i.e., neither from the users' point of view). In this section, we will show, which additional properties are required to make a NE a fair channel allocation.

We have seen in Section IV that in the selfish multi-radio channel allocation problem, the NE achieve load balancing. Unfortunately, these NE might be highly unfair by giving advantage to some players and neglecting others. For example, in the channel allocation presented in Figure 4 assuming that the total rate function $R^t(\cdot)$ is constant, player p_1 has the total rate $U_1 = \frac{5}{3}$, whereas player p_4 has the total rate $U_4 = \frac{4}{3}$. In order to study the fairness properties of the NE channel allocations, we use a particular metric called *max-min fairness (MMF)* as defined in [5]:

Definition 4: (Max-Min Fairness – MMF): The strategy matrix S^{mmf} is max-min fair if the payoff of player i cannot be increased without decreasing the payoff of another player j for which $U_i(S^{mmf}) \geq U_j(S^{mmf})$.

Using this concept, we identify the max-min fair NE channel allocations as expressed in Theorem 5.

Theorem 5: A NE channel allocation S^* is max-min fair if and only if $\sum_{c \in \mathcal{C}_{min}} k_{i,c} = \sum_{c \in \mathcal{C}_{min}} k_{j,c}$ for all $i, j \in \mathcal{N}$. This implies that $U_i = U_j, \forall i, j \in \mathcal{N}$.

In other words, if the total number of radios in the least allocated channels are equal for every player, the NE allocation is max-min fair. For example, the channel allocation in Figure 3 is max-min fair , but the one shown in Figure 4 is not.

From this theorem, we can immediately see that the perfectly balanced channel allocation is also max-min fair.

Corollary 1: If S^* is a NE such that $\mathcal{C}^- = \mathcal{C}^+$ (i.e., $k_c = k_d$, $\forall c, d \in \mathcal{C}$), then S^* is max-min fair as well.

VI. Coalition-proof Nash Equilibria

The definition of NE expresses the resistance to the deviation of a single player. In a realistic situation, it might be possible that several players collude to increase their payoff at the expense of other players. Such a collusion is called a *coalition*. The problem of how these coalitions are formed is a research topic in itself, thus in this paper we assume that any group of players can form a coalition. We can generalize the notion of NE for coalitions as defined in [4].

Definition 5: (Coalition-Proof Nash Equilibrium – CPNE): The strategy matrix S^{cpne} defines a coalition-proof Nash equilibrium if there *does not exist* any coalition $\Gamma \subset \mathcal{N}$ and any strategy of this coalition $S_\Gamma^{'}$ such that the following set of conditions is true:

$$U_i(S_\Gamma^{'}, S_{-\Gamma}^{cpne}) \geq U_i(S_\Gamma^{cpne}, S_{-\Gamma}^{cpne}), \forall i \in \Gamma \qquad (8)$$

with strict inequality for at least one player $i \in \Gamma$.

This means that no coalition can deviate from S^{cpne} such that the payoff of *at least one* of its members increases and the payoff of other members do not change.[3] From the definition, we can immediately see the following fact:

Players in a coalition can help each other in two ways. The first possibility is if a player relocates his radio to improve the payoff of another player he is in a coalition with. This property is expressed for two players in Lemma 2.

Lemma 2: If there exist two channels $c \in \mathcal{C}^+$ and $d \in \mathcal{C}^-$ and two players $i, j \in \mathcal{N}$ such that $k_{i,c} > 0$ and $k_{j,c} > 0$ whereas $k_{i,d} = 0$ and $k_{j,d} = 0$, then the NE channel allocation S^{cpne} is not coalition-proof.

The players in a coalition can also improve their payoff if they mutually remove some radios to reduce the number of radios contending for these channels. This property is shown for two players in Lemma 3.

Lemma 3: If there exist two channels $c, d \in \mathcal{C}^+$ or $c, d \in \mathcal{C}^-$ and two players $i, j \in \mathcal{N}$ such that $k_{i,c} > 0$, $k_{j,c} > 0$, $k_{i,d} > 0$ and $k_{j,d} > 0$, then the NE channel allocation S^{cpne} is not coalition-proof.

Based on Lemmas 2 and 3, we can prove the following theorem.

Theorem 6: If in a NE channel allocation S^* it is true that $|\mathcal{C}^+| \geq 2$, then S^* is not coalition-proof.

We narrowed down the set of potential coalition-proof Nash equilibria by Theorem 6. However, we could not find a set of sufficient conditions for coalition-proof Nash equilibria. Nevertheless, we can show that each coalition-proof NE is max-min fair as well. Note that the converse is not true.

Theorem 7: If NE channel allocation S^* is coalition-proof, then it is max-min fair as well.

As a summary, Figure 5 shows all channel allocations by properties.

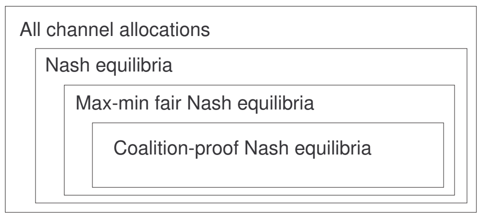

Fig. 5. Summary of channel allocations with different properties.

VII. Convergence to a Nash Equilibrium

We have demonstrated in Section IV that the non-cooperative behavior of the selfish players leads to load-balancing Nash equilibria. In this section, we propose two algorithms, each using a different set of available information, to enable the selfish players to converge to one of these Nash equilibria from an arbitrary initial configuration. The two algorithms are the following: 1) a centralized algorithm using perfect information and 2) a distributed algorithm using imperfect (local) information.

[3]Note that our definition corresponds to the principle of *weak deviation*. One can define the notion of a *strict deviation* of a coalition which requires that each coalition member increases its payoff by deviating. In the literature of coalition-proof equilibria, both concepts are used.

A. Centralized Algorithm Using Perfect Information

We have proved in Propositionr̃efprop:lem6-everywhere-no-diff2 that a Nash equilibrium channel allocation has a load-balancing property. In addition, we have shown in Theorem 3 that for a constant total rate function the Nash equilibria are Pareto-optimal as well. First we present the pseudo-code of Algorithm 1, a simple centralized algorithm to achieve one of these efficient Nash equilibria.

Algorithm 1 NE channel allocation with global coordination and perfect information

for $i = 1$ to $|\mathcal{N}|$ **do**
  for $j = 1$ to k **do**
    if $k_c = k_d, \forall c, d \in \mathcal{C}$ **then**
      use radio j on a channel c, where $k_{i,c} = 0$
    else
      use radio j on a channel c, where $k_c = \min_{d \in \mathcal{C}} k_d$
    end if
  end for
end for

Using the algorithm, the players allocate their radios such that they fill the channels almost equally. Note that the algorithm requires the sequential action of the players and hence it needs global coordination. In addition, the players must have perfect information about the number of radios on each of the channels. This can be achieved by the global coordination mentioned before or by having an extra radio per device for scanning the channels. Global coordination is unlikely to exist in a wireless networking scenario with selfish players. The second assumption about perfect information might not hold either, because selfish players should allocate all of their radios for communication as shown in Lemma 1. It is possible to model the cost of scanning with one radio instead of using it for communication. The investigation of this issue is part of our future work.

B. Distributed Algorithm Using Imperfect Information

In order to overcome the limitations of the centralized algorithm proposed in Section VII-A, we suggest a second algorithm that does not require global coordination and uses only imperfect information. In this subsection, we assume that players have imperfect information, meaning that they know the total number of radios on only those channels on which they operate a radio.

We define a *round-based* distributed algorithm that works as follows. First, we assume that there exists a random radio assignment of the players over the channels. For simplicity, we focus on the Nash equilibria that correspond to Theorem 1. This means that we assume that no player allocates more than one device on any channel. After the initial channel assignment, each player tries to improve his total throughput by reorganizing his radios. To avoid that all players change together, we leverage the technique of backoff mechanism well known in the IEEE 802.11 medium access technology [24]. We define a *backoff window* BW and each player chooses a random initial value for his *backoff counter* with uniform probability from the set $\{1, \ldots, BW\}$. Then in every round each player decreases his backoff counter by one and applies the re-allocation of his radios only when the backoff counter reaches zero. After he changes his channel allocation, he resets the backoff counter as described previously. We can notice that using the backoff mechanism, the players play a game in an almost sequential order.

In each round, when player i's backoff counter is equal to zero, he calculates the average number of devices on the channels he knows (recall that we denote this set by $\mathcal{C}_i$). We denote the average number of devices on the channels in $\mathcal{C}_i$ by K_i. For each channel $d \in \mathcal{C}_i$ with $k_d - K_i \geq 1$ player i *moves his radio to another channel* $c \notin \mathcal{C}_i$. The probability to chose a channel $c \notin \mathcal{C}_i$ is $\frac{1}{|\mathcal{C} \setminus \mathcal{C}_i|}$. This is the first property of the algorithm with imperfect information.

We can show that the above procedure reaches a stable state. Unfortunately, the available local information might be insufficient for the players to determine if the achieved stable state is Nash equilibrium. We show an example for such a "false Nash equilibrium" in Figure 6.

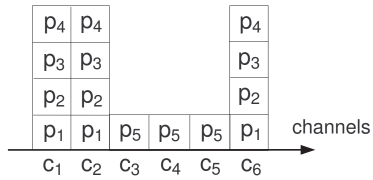

Fig. 6. An example for a stability state using the distributed algorithm with imperfect information. Here $|\mathcal{C}| = 6$, $|\mathcal{N}| = 5$ and $k = 3$. Each player believes that this is a Nash equilibrium due to the insufficient local information.

In order to solve the problem of inefficient stable states, we introduce the following mechanism: player i checks the number of radios for each of the channels $d \in \mathcal{C}_i$ as suggested above and *with a small probability* ϵ *he moves his radio to another channel* $c \notin \mathcal{C}_i$ even if $0 < k_d - K_i < 1$. He chooses the new channel c with a probability $\frac{1}{|\mathcal{C} \setminus \mathcal{C}_i|}$ as presented before. This second property allows us to resolve the inefficient stability states, but at the same time, it will also cause the instability of the Nash equilibria.

We provide the description of our algorithm below. Note that this algorithm includes both properties: 1) the backoff mechanism to randomize the changes and 2) the mechanism to resolve inefficient stable states.

Due to the second property of our algorithm, it does not perfectly converge to the existing Nash equilibria (more precisely, it converges there with high probability, but it does not stay in a Nash equilibrium solution). Nevertheless, we can observe that the algorithm remains in states that are "close" to Nash equilibria in terms of load-balancing. We demonstrate this intuition by the simulations presented in Section VII-C.

C. Simulation Results for Algorithm 2

We implemented Algorithm 2 in MATLAB and with a special focus on wireless IEEE 802.11a protocol (meaning that we have chosen 8 orthogonal channels as a default value for $|\mathcal{C}|$). In this subsection, we present our simulation results showing the convergence time and efficiency of Algorithm 2. In each of the simulations, we assume a constant total rate

Algorithm 2 Distributed NE channel allocation algorithm using local information

```
random channel allocation
while () do
  get the current channel allocation
  for i = 1 to |N| do
    if backoff counter is 0 then
      if (max_{c∈C_i}(k_c) − min_{c∈C_i}(k_c) > 1) then
        for j = 1 to k do
          assume that radio j uses channel d
          if k_d > K_i then
            move the radio j from d to c ∉ C_i, where c is chosen
            with uniform random probability from the set C\C_i
          end if
        end for
      else
        for j = 1 to k do
          assume that radio j uses channel d
          if k_d ≥ K_i then
            move the radio j from d to c ∉ C_i with probability
            ε, where c is chosen with uniform random probability
            from the set C\C_i
          end if
        end for
      end if
      reset the backoff counter to a new value from the set
      {1, ..., BW}
    else
      decrease the backoff counter value by one
    end if
  end for
end while
```

function $R^t(\cdot)$. Note however, that the algorithm shows similar properties for any decreasing total rate function introduced in Section III.

Let us first highlight the best and worst case in terms of the desired load-balancing for Algorithm 2. The best case is one of the NE channel allocations, and the worst case is characterized by the fact that there exist k channels where each of the players have a radio, whereas the rest of the channels have no radios at all. In Figure 7, we present an example of the worst case channel allocation that is opposed to the best case NE in Figure 3 for $|\mathcal{C}| = 6$ and we refer to it as *unbalanced (UB)* channel allocation.

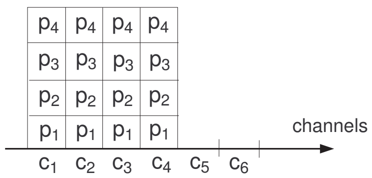

Fig. 7. An example for a worst case channel allocation for Algorithm 2. The channel allocation is completely unbalanced, as opposed to the NE (best case) shown in Figure 3. Here $|\mathcal{C}| = 6$, $|\mathcal{N}| = 4$ and $k = 4$.

We calculate the average number of radios per channel as $m = \frac{|\mathcal{N}|\cdot k}{|\mathcal{C}|}$. We can compare the utilization of every channel c to the average to achieve the total balance of the channel allocation S:

Definition 6: (Balance:) The *balance* β of a channel allocation S is defined as the sum $\beta(S) = \sum_{c\in|\mathcal{C}|} |k_c - m|$.

The notion of balance allows us to define the efficiency of a given channel allocation as a proportion between the worst case and the best case channel allocations.

Definition 7: (Efficiency:) The *efficiency* ϕ of a channel allocation S is defined as $\phi(S) = \frac{\beta(S_{\mathrm{UB}})-\beta(S)}{\beta(S_{\mathrm{UB}})-\beta(S_{\mathrm{NE}})}$.

Let us emphasize that for any channel allocation S, it is true that $0 \leq \phi(S) \leq 1$. Furthermore, $\phi(S_{\mathrm{NE}}) = 1$ and $\phi(S_{\mathrm{UB}}) = 0$ as desired by this measure.

Let us now define the *average efficiency* over time and the *efficiency ratio*. To this end, we denote the efficiency in round t by $\phi(t, S)$.

Definition 8: (Average efficiency and efficiency ratio:) The *average efficiency* $\overline{\phi}$ at round T is defined as the sum $\overline{\phi}(T, S) = \sum_{t=1}^{T} \phi(t, S)$. We define the *efficiency ratio* as $\Phi = \liminf_{T\to\infty} \frac{\overline{\phi}(T,S)}{T}$.

Note that the efficiency ratio expresses the performance of the distributed channel allocation algorithm per round over a long period of time. In our simulations, we applied a finite simulation time, hence we measured the efficiency ratio for $T = 10000$ rounds.

Finally, let us define the *convergence time* of Algorithm 2 as follows.

Definition 9: (Convergence Time): We define the *convergence time* of Algorithm 2, as the time when the channel allocation efficiency first reaches the value of one (i.e., the efficiency of a NE, $\phi(S_{\mathrm{NE}})$).

We assume that the duration of one round in the updating algorithm is 10ms. This duration of one round corresponds roughly to the time needed for all these devices to transmit one MAC layer packet, i.e., the time that the devices can learn about other devices in the channel. As mentioned previously, we run each simulation for 10000 rounds, which corresponds to 100s according to the assumption above. Each average value is the result of 100 simulation runs. For the convergence time simulations, we present our results with a 0.95 confidence level on the mean value.

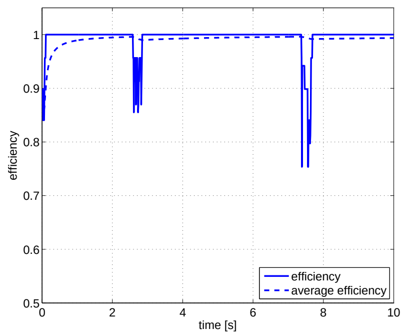

Fig. 8. One simulation run: Efficiency and averaged efficiency vs. time using the values $|\mathcal{C}| = 8$, $|\mathcal{N}| = 10$, $k = 3$ and $W = 15$.

Let us first present an example run for our distributed algorithm with imperfect information in Figure 8 for 20s. One can notice that the algorithm quickly reaches the NE state and thus the average efficiency converges to one. Also, one can observe that the players sometimes leave the NE state due to the second property (change a radio on a channel $c \in \mathcal{C}_{max}$ in a stable state with probability ϵ), but they quickly return to it.

Suppose that the total data rate per channel is $R^t(k_c) = 54Mbps$, for any k_c. In Figure 9, we present a snapshot of the total payoff for the players in one of the NE reached in the previous simulation. One can observe that the total payoff is very similar for the players, hence we conclude that our algorithm converges to fair channel allocations.

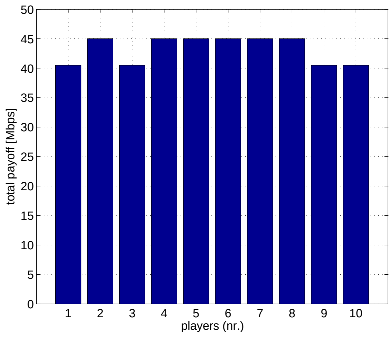

Fig. 9. Total payoff received by each device in the NE channel allocation. We have the parameter values $|\mathcal{C}| = 8$, $|\mathcal{N}| = 10$ and $k = 3$.

Next, we investigate the effect of the number of radios per device on the efficiency ratio (shown in Figure 10a) and on the convergence time of the algorithm (presented in Figure 10b). We can observe on the figures that Algorithm 2 converges fast with high efficiency ratio if the number of radios per device is 3 or 5. The higher the number of radios per device, the more channels the players know. Hence, more information helps them making their decisions. This is the reason that the convergence is slower if the number of radios is 2. For two radios per devices, the effect of changing the channel for even one radio has a significant impact that undermines the stability of the NE more easily. If the number of radios is 4, then convergence is slow for another reason: There is only one Nash equilibrium channel allocation, namely the perfectly flat one; thus it takes more time to find it. With longer convergence, the efficiency ratio decreases as well.

In Figures 10c and 10d, we investigate the effect of the number of players, each device having three radios. We can see that our distributed algorithm keeps the system in an efficient state, although the efficiency is slightly lower for multiples of $|\mathcal{N}|$ with higher convergence time. As mentioned above, the reason is that in this case, there exists only one NE (the perfectly load-balanced) and thus it is more difficult to converge to.

In summary, we can observe that, in spite of the fact that convergence is not theoretically ensured, the proposed distributed algorithm based on imperfect information ensures high system performance and good convergence time.

VIII. Conclusion

In this paper, we have considered the problem of competitive channel allocation among devices that use multiple radios. Our main conclusion is that, in spite of the non-cooperative behavior of such devices, their Nash equilibrium channel allocations result in load balancing. We have also studied fairness issues and coalition-proof NE. Finally, we have provided three algorithms to achieve the efficient, load-balancing Nash equilibrium channel allocation and we have studied their convergence properties either theoretically or numerically.

In terms of future work, we will extend our current model to include different channel characteristics and payoffs. We will also pursue our theoretical investigations of selfish multi-radio channel allocation for general topologies. We will pay particular attention to the application of existing fairness metrics in the competitive context. In addition, we will take the cost of channel scanning into consideration. Last but not least, we will study convergence algorithms that achieve Nash equilibria in general topologies.

References

[1] A. Adya, P. Bahl, J. Padhye, A. Wolman, and L. Zhou. A multi-radio unification protocol for IEEE 802.11 wireless networks. In *Proceedings of Broadnets'04*, pages 344–354, Lausanne, Switzerland, 2004.
[2] I. F. Akyildiz, X. Wang, and W. Wang. Wireless mesh networks: A survey. *Computer Networks, Elsevier North-Holland, Inc.*, 47:445–487, Mar. 2005.
[3] M. Alicherry, R. Bhatia, and L. (Erran) Li. Joint channel assignment and routing for throughput optimization in multi-radio wireless mesh networks. In *Proceedings of ACM/IEEE International Conference on Mobile Computing and Networking (Mobicom'05)*, pages 58–72, Cologne, Germany, Aug. 28 - Sep. 2 2005.
[4] B. D. Bernheim, B. Peleg, and M. D. Whinston. Coalition-proof Nash equilibria: I concepts. *Journal of Economic Theory*, 42(1), Jun. 1987.
[5] D. Bertsekas and R. Gallager. *Data Networks*. Prentice-Hall, 2nd ed., 1992.
[6] G. Bianchi. Performance analysis of the IEEE 802.11 distributed coordination function. *IEEE Journal on Selected Areas in Communication*, 18(3), Mar. 2000.
[7] M. Cagalj, S. Ganeriwal, I. Aad, and J.-P. Hubaux. On selfish behavior in CSMA/CA networks. In *Proceedings of the IEEE Conference on Computer Communications (INFOCOM '05)*, Miami, USA, Mar. 13-17 2005.
[8] L. Cao and H. Zheng. Distributed spectrum allocation via local bargaining. In *Proceedings of IEEE SECON'05)*, Santa Clara, California, USA, Sep. 26-29 2005.
[9] M. M. L. Cheng and J. C. I. Chuang. Performance evaluation of distributed measurement-based dynamic channel assignment in local wireless communications. *IEEE Journal on Selected Areas in Communications*, 14(4):698–710, May 1996.
[10] D. C. Cox and D. O. Reudink. Dynamic channel assignment in two dimension large-scale mobile radio systems. *Bell Sys. Tech.*, 51:1611–1628, 1972.
[11] M. Félegyházi. *Non-cooperative Behavior in Wireless Networks*. PhD thesis, EPFL – Switzerland, Apr. 2007.
[12] M. Félegyházi and J.-P. Hubaux. Game theory in wireless networks: A tutorial. Technical Report LCA-REPORT-2006-002, EPFL, Feb. 2006.
[13] D. Fudenberg and J. Tirole. *Game Theory*. MIT Press, 1991.
[14] M. M. Halldórsson, J. Y. Halpern, L. (Erran) Li, and V. S. Mirrokni. On spectrum sharing games. In *Proceedings of the 23rd annual ACM symposium on Principles of Distributed Computing (PODC 2004)*, pages 107–114, St. John's, Newfoundland, Canada, Jul. 25-28 2004.
[15] S. Haykin. Cognitive radio: Brain-empowered wireless communications. *IEEE Journal on Selected Areas in Communications (JSAC)*, 23(2):201–220, Feb. 2005.
[16] I. Katzela and M. Naghshineh. Channel assignment schemes for cellular mobile telecommunication systems: a comprehensive survey. *IEEE Personal Communications*, 3(3):10–31, Jun. 1996.
[17] J. Konorski. Multiple access in ad-hoc wireless LANs with noncooperative stations. In *Proceedings of Networking 2002*, pages 1141–1146, 2002.
[18] E. Koutsoupias and C. Papadimitriou. Worst-case equilibria. In *Proceedings of the 16th Annual Symposium on Theoretical Aspects of Computer Science (STACS'99)*, Mar. 1999.
[19] A. B. MacKenzie and S. B. Wicker. Stability of multipacket slotted aloha with selfish users and perfect information. In *Proceedings of the IEEE Conference on Computer Communications (INFOCOM '03)*, San Francisco, USA, Mar. 30 - Apr. 3 2003.

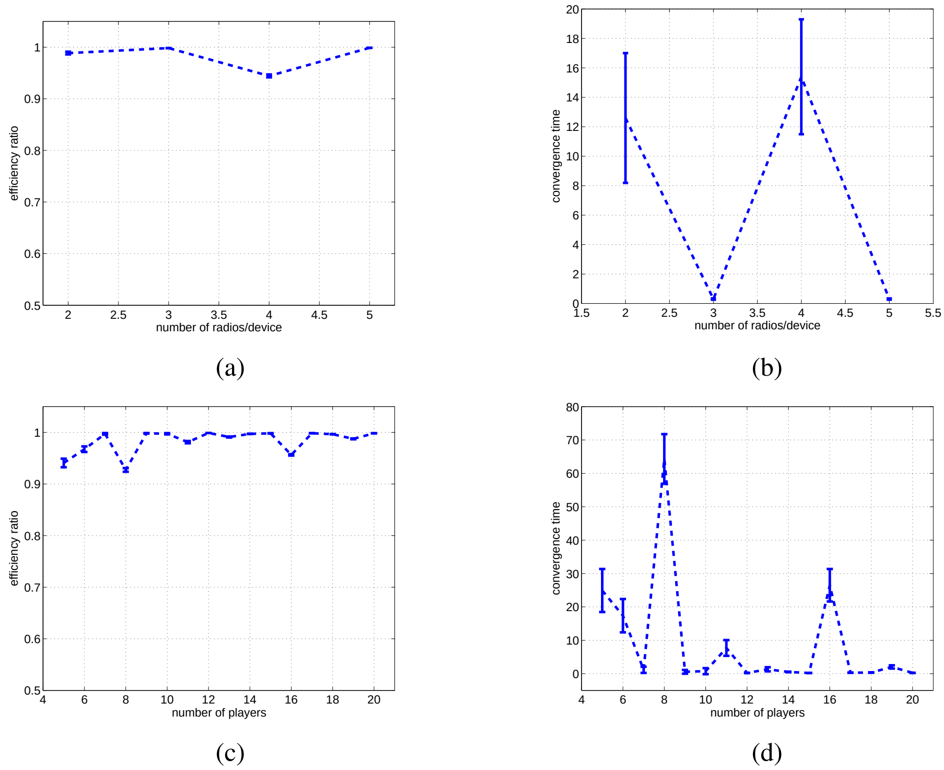

Fig. 10. The effect of the total number of radios: (a) The efficiency ratio and (b) the convergence time as a function of the number of radios per device k. Similarly, we show (c) the efficiency ratio and (d) the convergence time as a function of the number of players $|\mathcal{N}|$. The simulation parameters are $|\mathcal{C}| = 8$, $\epsilon = 10^{-4}$ and $W = 15$. In addition, we used the following default values $|\mathcal{N}| = 10$ and $k = 3$, where they did not correspond to the measured parameter.

[20] A. Mishra, S. Banerjee, and W. Arbaugh. Weighted coloring based channel assignment for WLANs. *Mobile Computing and Communications Review (MC2R)*, 9(3), 2005.

[21] N. Nie and C. Comaniciu. Adaptive channel allocation spectrum etiquette for cognitive radio networks. In *Proceedings of IEEE Dynamic Spectrum Access Networks (DySPAN'05)*, Baltimore, USA, Nov. 2005.

[22] A. Raniwala and T.-C. Chiueh. Architecture and algorithms for an IEEE 802.11-based multi-channel wireless mesh network. In *Proceedings of the IEEE Conference on Computer Communications (INFOCOM '05)*, Miami, USA, Mar. 13-17 2005.

[23] T. S. Rappaport. *Wireless Communications: Principles and Practice (2nd Edition)*. Prentice Hall, 2002.

[24] M. Schwartz. *Mobile Wireless Communications*. Cambridge Univ. Press, 2005.

[25] J. van den Heuvel, R. A. Leese, and M. A. Shepherd. Graph labeling and radio channel assignment. *Journal of Graph Theory*, 29:263–283, 1998.

[26] J. Zander and H. Eriksson. Asymptotic bounds on the performance of a class of dynamic channels assignment algorithms. *IEEE Journal on Selected Areas in Communications (JSAC)*, 11(6):926–933, Aug. 1993.

[27] H. Zheng and L. Cao. Device-centric spectrum management. In *Proceedings of IEEE DySPAN'05)*, Baltimore, USA, Nov. 8-11 2005.